C H A P T E R 5

MAGNETISM

5.1 INTRODUCTION

1. *What do you mean by the terms magnet and magnetism ? What are natural magnets ? What is the origin of the word magnetism ?*

Magnets and magnetism. *A magnet is a material that has both attractive and directive properties.* It attracts small pieces of iron, nickel, cobalt, etc. This property of attraction is called *magnetism*. When suspended freely, a thin long piece of magnet comes to rest nearly in the geographical north-south direction.

As early as the 6th century B.C., the Greeks had some knowledge of ***natural magnets***. *Thales of Miletus* knew that pieces of a naturally occurring iron ore, *lodestone or magnetite* or black iron oxide Fe_3O_4 had the property of attracting small pieces of iron. The word *magnetism* originates from the place – *Magnesia* – a province in the upper part of Greece – where this ore was found. Later on the Chinese discovered that thin long pieces of lodestone, if suspended horizontally and freely with a string, would naturally orient themselves roughly in the geographical north-south direction. In fact, the word lodestone means *a leading stone* which expresses this directional property, valuable to travellers. By about A.D. 1000, the Chinese were using the magnetic compass for navigation. It was *William Gilbert* who, in his book ***'De Magnete'*** of 1600, first suggested that the earth itself was a huge magnet, causing the alignment of compass needles.

In the early part of the nineteenth century, *Oersted* discovered that moving charges or currents are the sources of magnetic fields. However, the science of magnetism was known long before the nineteenth century.

5.2 ARTIFICIAL MAGNETS

2. *What are artificial magnets ? What are their common shapes ?*

Artificial magnets. Generally, the natural magnets are not strong enough magnetically and have

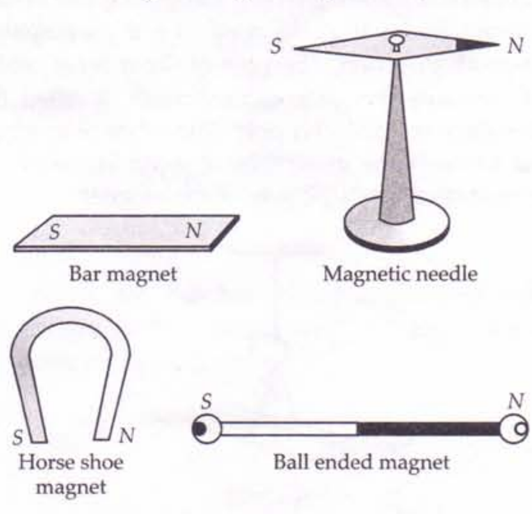


Fig. 5.1 Different forms of artificial magnets.

inconvenient shapes. The *pieces of iron and other magnetic materials can be made to acquire the properties of natural magnets. Such magnets are called* ***artificial magnets.*** The main advantage of these magnets is that they can be made much more stronger than the natural magnets and also of any convenient shape and size. They are generally available in the following forms :

1. Bar magnet. It is a bar of circular or rectangular cross-section.

2. Magnetic needle. It is a thin magnetised steel needle having pointed ends and is pivoted at its centre so that it is free to rotate in a horizontal plane.

3. Horse shoe magnet. It has the shape of a horse-shoe and thus it has been named so.

4. Ball-ended magnet. It is a thin bar of circular cross-section ending in two spherical balls.

5.3 BASIC PROPERTIES OF MAGNETS

3. *State the important properties of magnets.*

Basic properties of magnets :

1. ***Attractive property***. A magnet attracts small pieces of iron, cobalt, nickel, etc. When a magnet is brought near a heap of iron filings, the ends of the magnet show the greatest attraction. These ends, where the magnetic attraction is the maximum, are called *poles* of the magnet. Thus *every magnet has two poles*.

Fig. 5.2 Poles of a bar magnet.

2. ***Directive property***. When a magnet is suspended or pivoted freely, it aligns itself in the geographical north-south direction. The pole of the magnet which points towards the geographical north is called the *north-seeking* or north (N) pole. The other pole which points towards the geographical south is called the *south-seeking* or south (S) pole of the magnet.

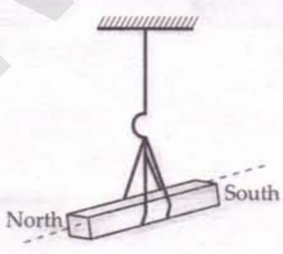


Fig. 5.3 A magnet points north-south when freely suspended.

3. ***Like poles repel and unlike poles attract***. If the N-pole of a magnet is brought near the N-pole of a suspended magnet, the poles are found to repel each other. Two S-poles also repel each other. In contrast, N- and S- poles always attract each other. This action can be described by the ***law of magnetic poles*** which states that *like magnetic poles repel, and unlike magnetic poles attract each other*.

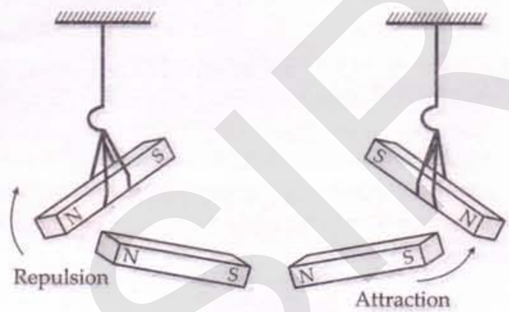


Fig. 5.4 Like poles repel and unlike poles attract.

4. ***Magnetic poles always exist in pairs***. If we try to isolate the two poles of a magnet from each other by breaking the magnet in the middle, each broken part is found to be a magnet with N- and S-poles at its ends. If we break these parts further, each part again is found to be a magnet. So unlike electric charges, *magnetic monopoles do not exist*. Every magnet *exists as a dipole*.

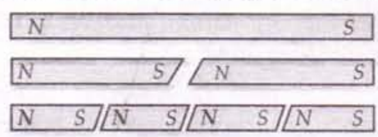


Fig. 5.5 Poles always exist in pairs.

5. ***Magnetic induction***. A magnet induces magnetism in a magnetic substance placed near it. This phenomenon is called *magnetic induction*. When N-pole of a powerful magnet is placed close to a soft iron bar, the closer end of the bar becomes S-pole and the farther end N-pole. As a result, the magnet attracts the iron bar. Thus *induction precedes attraction*.

For Your Knowledge

- **Repulsion is the surer test of magnetism.** A magnet can attract another magnet. Also it can attract magnetic substances like iron, nickel, cobalt, etc. However, a magnet can repel another magnet only. So repulsion is the surer test of magnetism.

5.4 SOME IMPORTANT DEFINITIONS CONNECTED WITH MAGNETISM

4. *Define the terms magnetic field, uniform magnetic field, magnetic poles, magnetic axis, magnetic equator and magnetic length with reference to a bar magnet.*

Some important definitions connected with magnetism :

1. Magnetic field. *The space around a magnet within which its influence can be experienced is called its magnetic field.*

2. Uniform magnetic field. *A magnetic field in a region is said to be uniform if it has same magnitude and direction at all points of that region.* At a given place, the magnetic field of the earth can be considered uniform. The field due to a bar magnet is not uniform.

A uniform magnetic field acting in the plane of paper is represented by equidistant parallel lines [Fig. 5.6(*a*)]. A uniform magnetic field acting perpendicular to the paper and directed outwards is represented by dots [Fig. 5.6(*b*)]. A uniform magnetic field acting perpendicular to the plane of paper and directed inwards is represented by crosses [Fig. 5.6(*c*)].

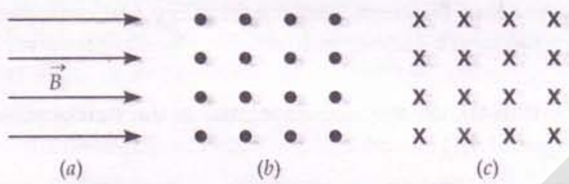


Fig. 5.6 Representations of a uniform magnetic field.

3. Magnetic poles. *These are the regions of apparently concentrated magnetic strength in a magnet where the magnetic attraction is maximum.* The poles of a magnet lie somewhat inside the magnet and not at its geometrical ends.

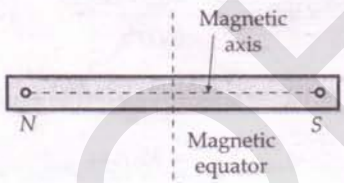


Fig. 5.7 A bar magnet.

4. Magnetic axis. *The line passing through the poles of a magnet is called the magnetic axis of the magnet.*

5. Magnetic equator. *The line passing through the centre of the magnet and at right angles to the magnetic axis is called the magnetic equator of the magnet.*

6. Magnetic length. *The distance between the two poles of a magnet is called the magnetic length of the magnet. It is slightly less than the geometrical length of the magnet.*

It is found that $\dfrac{\text{Magnetic length}}{\text{Geometrical length}} = 0.84$

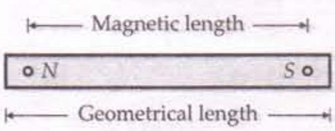


Fig. 5.8 Magnetic and geometrical lengths of a magnet.

5.5 COULOMB'S LAW OF MAGNETIC FORCE

5. *State Coulomb's law of magnetic force. Hence define a unit magnetic pole.*

Coulomb's law of magnetic force. *This law states that the force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.*

If q_{m_1} and q_{m_2} are the pole strengths of the two magnetic poles which are distance r apart, then the force between them is given by

$$F \propto \frac{q_{m_1} q_{m_2}}{r^2}$$

or

$$F = k \cdot \frac{q_{m_1} q_{m_2}}{r^2}$$

where k is a proportionality constant which depends on the nature of the medium as well as on the system of units chosen. For SI units and for poles in vacuum,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{q_{m_1} q_{m_2}}{r^2}$$

where μ_0 is the permeability of free space and is equal to $4\pi \times 10^{-7}$ henry/metre. We can define unit magnetic pole from Coulomb's law :

If $q_{m_1} = q_{m_2} = 1$ unit ; $r = 1$ m, then

$$F = \frac{\mu_0}{4\pi} = 10^{-7}\ \text{N}$$

Hence a ***unit magnetic pole*** *may be defined as that pole which when placed in vacuum at a distance of one metre from an identical pole repels it with a force of* 10^{-7} *newton.*

5.6 MAGNETIC DIPOLE AND MAGNETIC DIPOLE MOMENT

6. *What is a magnetic dipole ? Give some examples.*

Magnetic dipole. In electricity, the fundamental or simplest structure that can exist is a *point charge.* Here two equal and opposite charges separated by a small distance constitute an *electric dipole,* which is described by an electric dipole moment $\vec{p}$. In magnetism, isolated magnetic poles do not exist. Here the simple structure that can exist is the *magnetic dipole* which is described by a magnetic dipole moment $\vec{m}$.

An arrangement of two equal and opposite magnetic poles separated by a small distance is called a magnetic dipole.

Every bar magnet is a magnetic dipole. A current carrying loop behaves as a magnetic dipole. Even an atom acts as a magnetic dipole due to the circulatory motion of the electrons around its nucleus.

7. *Define magnetic dipole moment. Is it a scalar or vector quantity ? Give its SI unit.*

Magnetic dipole moment. The *magnetic dipole moment* of a magnetic dipole is defined as the product of its pole strength and magnetic length. It is a vector quantity, directed from S-pole to N-pole.

$$\vec{m} = q_m \times 2\vec{l}$$

where q_m is the pole strength and $2\vec{l}$ is the magnetic length of the dipole measured in the direction S – to N – pole.

We shall see later on that the SI unit of magnetic dipole moment is **ampere metre^2 (Am^2)** or **joule per tesla (JT^{-1})**.

For Your Knowledge

- Basic difference between electricity and magnetism. In electricity a point charge is the *simplest source* of electricity and can be used as a *test object* also. By measuring the force on a test charge at various places in a given electric field, we can map out the entire field. In magnetism, on the other hand, *isolated magnetic poles do not exist*. A *magnetic dipole* is the simplest source of a magnetic field. It can be used as a test object for mapping a magnetic field. The mapping is done by measuring the torque experienced by a test magnet or a magnetic dipole at various points in the magnetic field.

  In short, a small magnet is described by a *vector* $\vec{m}$ while an electric charge by the *scalar* charge q on it. A magnet experiences a torque in a magnetic field while an electric charge experiences a force in an electric field.
- The pole strength q_m is also called *magnetic charge*. Thus we assign magnetic charge $+q_m$ to the north pole and $-q_m$ to the south pole.
- The direction of magnetic dipole moment $\vec{m}$ is from S-pole to N-pole. This is analogous to the direction of the electric dipole moment $\vec{p}$ of an electric dipole from negative charge to positive charge.
- When a magnet of pole strength q_m is cut into two equal parts.

  (*i*) along its axis (longitudinally), the pole strength of each half becomes $q_m/2$.

  (*ii*) perpendicular to its axis (transversely), the pole strength of each half still remains q_m.

Examples based on Coulomb's Law and Dipole Moment of a Magnet

Formulae Used

1. Magnetic dipole moment, $m = q_m \times 2l$
2. Coulomb's law, $F = \frac{\mu_0}{4\pi} \cdot \frac{q_{m_1} q_{m_2}}{r^2}$

Units Used

Pole strength is in Am, force in newton, distance in metre.

Constant Used

$\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}.

Example 1. *Two magnetic poles, one of which is four times stronger than the other, exert a force of 5 g f on each other when placed at a distance of 10 cm. Find the strength of each pole.*

Solution. Let the pole strengths of the two dipoles be q_m and $4q_m$.

Here $F = 5\text{ g f} = 5 \times 10^{-3}\text{ kg f} = 5 \times 10^{-3} \times 9.8\text{ N}$, $r = 10\text{ cm} = 0.1\text{ m}$

Using Coulomb's law of magnetism,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{q_{m_1} q_{m_2}}{r^2}$$

$$\therefore \quad 5 \times 10^{-3} \times 9.8 = \frac{10^{-7} \times q_m \times 4q_m}{(0.1)^2}$$

or $$q_m^2 = \frac{5 \times 9.8 \times (0.1)^2 \times 10^4}{4} = 25 \times 49$$

or $$q_m = 5 \times 7 = \mathbf{35\ Am}$$

and $$4q_m = 4 \times 35 = \mathbf{140\ Am}.$$

Example 2. *Two similar magnetic poles, having pole strengths in the ratio 1 : 2 are placed 1 m apart. Find the point where a unit pole experiences no net force due to the two poles.*

Solution. Let the pole strengths of the two magnetic poles be q_m and $2q_m$. Suppose the required point is located at distance x from the first pole. Then at this point,

Force on unit pole due to first pole

= Force on unit pole due to second pole

or $$\frac{\mu_0}{4\pi} \cdot \frac{q_m \times 1}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{2\,q_m \times 1}{(1-x)^2}$$

or $$2x^2 = (1-x)^2 \quad \text{or} \quad \sqrt{2}x = 1 - x$$

or $$x = \frac{1}{1+\sqrt{2}} = \mathbf{0.414\ m.}$$

Example 3. *Calculate the force acting between two magnets of length 15 cm each and pole strength 80 Am each when the separation between their north poles is 10 cm and that between south poles is 40 cm.*

Solution. The situation is shown in Fig. 5.9. Here $q_{m_1} = q_{m_2} = 80$ Am

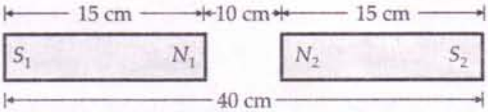


Fig. 5.9

Force of repulsion between poles N_1 and N_2 is

$$F_1 = \frac{\mu_0}{4\pi} \cdot \frac{q_{m_1} q_{m_2}}{r^2} = \frac{10^{-7} \times 80 \times 80}{(0.10)^2} = 0.064 \text{ N}$$

Force of repulsion between poles S_1 and S_2 is

$$F_2 = \frac{10^{-7} \times 80 \times 80}{(0.40)^2} = 0.004 \text{ N}$$

Force of attraction between N_1 and S_2 is

$$F_3 = \frac{10^{-7} \times 80 \times 80}{(0.25)^2} = 0.010 \text{ N}$$

Force of attraction between N_2 and S_1 is

$$F_4 = F_3 = 0.010 \text{ N}$$

Resultant force between the two magnets is

$$F = F_1 + F_2 - F_3 - F_4$$
$$= 0.064 + 0.004 - 0.010 - 0.010$$
$$= \mathbf{0.048 \text{ N}} \text{ (repulsive).}$$

Example 4. *A magnetic dipole of length 10 cm has pole strength of 20 Am. Find the magnetic moment of the dipole.*

Solution. Here $2l = 10 \text{ cm} = 0.10 \text{ m}$, $q_m = 20$ Am

∴ Magnetic moment,

$$m = q_m \times 2l = 20 \times 0.10 \text{ Am}^2 = \mathbf{2.0 \text{ Am}^2}.$$

Example 5. *A bar magnet of magnetic moment 5.0 Am^2 has poles 20 cm apart. Calculate the pole strength.* [CBSE D 92C]

Solution. Here $m = 5.0 \text{ Am}^2$, $2l = 20 \text{ cm} = 0.20$ m

As $m = q_m \times 2l$

∴ Pole strength,

$$q_m = \frac{m}{2l} = \frac{5.0}{0.20} = \mathbf{25 \text{ Am.}}$$

Example 6. *A steel wire of length l has a magnetic moment m. It is bent into a semicircular arc. What is the new magnetic moment ?*

Solution. Pole strength, $q_m = \frac{m}{l}$

When the wire is bent into a semicircular arc, the separation between the poles changes from l to $2r$, where r is the radius of the semicircular arc. Thus

$$l = \pi r \quad \text{or} \quad r = \frac{l}{\pi}$$

$$\text{New magnetic moment} = q_m \times 2r = \frac{m}{l} \times \frac{2l}{\pi} = \frac{\mathbf{2m}}{\boldsymbol{\pi}}.$$

Problems For Practice

1. Two magnetic south poles are located 4.0 cm apart. If the poles of each magnet have a strength of 8.0 Am and are 20.0 cm apart, find the force exerted by one south pole on the other. (Ans. 4.0×10^{-3} N)
2. Two equal and unlike poles placed 5 cm apart in air attract each other with a force of 14.4×10^{-4} N. How far from each other should they be placed so that the force of attraction will be 1.6×10^{-4} N ? (Ans. 0.15 m)
3. Two magnetic poles, one of which is 10 times as strong as the other, exert on each other a force equal to 9.604 mN, when placed 10 cm apart in air. Find the strength of the two poles. (Ans. 9.8 Am, 98 Am)
4. Two like magnetic poles of strengths 5 Am and 20 Am are situated 1.0 m apart. At what point on the line joining the two poles, will the magnetic field be zero ? (Ans. 0.33 m from 5 Am pole towards 20 Am pole, 1 m from 5 Am pole away from 20 Am pole)
5. Two bar magnets of length 0.1 m and pole strength 75 Am each, are placed on the same line. The distance between their centres is 0.2 m. What is the resultant force due to one on the other when (*i*) the north pole of one faces the south pole of the other and (*ii*) the north pole of one faces the north pole of the other ? [Ans. (*i*) 3.4×10^{-2} N (attraction), (*ii*) 3.4×10^{-2} N (repulsion)]
6. A magnetic dipole of length 15 cm has a dipole moment of 1.5 Am^2. What is the pole strength ? (Ans. 10 Am)
7. A magnetised steel wire 31.4 cm long has a pole strength of 0.2 Am. It is bent in the form of a semicircle. Calculate the magnetic moment of the steel wire. (Ans. 0.04 Am^2)
8. Two thin bar magnets of pole strengths 25 Am and 48 Am respectively and lengths 0.20 m and 0.25 m respectively are placed at right angles to each other with the N-pole of first touching the S-pole of the second. Find the magnetic moment of the system. (Ans. 13 Am^2)

HINTS

1. $F = \frac{\mu_0}{4\pi} \cdot \frac{q_{m_1} q_{m_2}}{r^2} = \frac{10^{-7} \times 8 \times 8}{(4 \times 10^{-2})^2}$ N $= \mathbf{4.0 \times 10^{-3}}$ **N.**

2. $F = \frac{\mu_0}{4\pi} \frac{q_{m_1} q_{m_2}}{r^2}$ *i.e.,* $F \propto \frac{1}{r^2}$

$\therefore \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$

$r_2 = \sqrt{\frac{F_1}{F_2}} \cdot r_1 = \sqrt{\frac{14.4 \times 10^{-4}}{1.6 \times 10^{-4}}} \times 5 \times 10^{-2} = \mathbf{0.15\ m}.$

3. As $F = \frac{\mu_0}{4\pi} \cdot \frac{q_{m_1} q_{m_2}}{r^2}$

$\therefore \quad 9.604 \times 10^{-3} = \frac{10^{-7} \times q_m \times 10\, q_m}{(0.1)^2}$

or $q_m^2 = 96.04$

or $q_m = \mathbf{9.8\ Am}$

and $10\, q_m = \mathbf{98\ Am}.$

4. Let the magnetic field be zero at distance x from the pole of strength 5 Am.

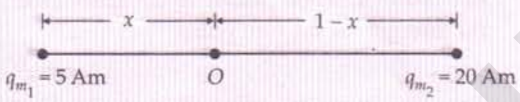


Fig. 5.10

Magnetic field at O due to 5 Am pole

= Magnetic field at O due to 20 Am pole

$\frac{\mu_0}{4\pi} \cdot \frac{5}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{20}{(1-x)^2}$

or $4x^2 = (1-x)^2$

or $2x = \pm(1-x)$

or $x = \mathbf{-1\ m, +0.33\ m}.$

5. Proceed as in Example 2 on page 5.4.

6. $q_m = \frac{m}{2l} = \frac{1.5\ \text{Am}^2}{0.15\ \text{m}} = \mathbf{10\ Am}.$

7. Proceed as in Example 6 on page 5.5.

8. Here $m_1 = 25 \times 0.20 = 5.0\ \text{Am}^2$

$m_2 = 48 \times 0.25 = 12.0\ \text{Am}^2$

Resultant magnetic moment of the two magnets placed perpendicular to each other is

$m = \sqrt{m_1^2 + m_2^2} = \sqrt{5^2 + 12^2} = \mathbf{13\ Am^2}.$

5.7 MAGNETIC FIELD LINES

8. *What are magnetic lines of force ? Give their important properties.*

Magnetic lines of force. *Michael Faraday,* the celebrated physicist of London (1791–1867) introduced the concept of the magnetic lines of force to represent a magnetic field *visually.* Magnetic lines of force do not really exist but they are quite useful in describing many different magnetic phenomena.

A magnetic line of force *may be defined as the curve the tangent to which at any point gives the direction of the magnetic field at that point. It may also be defined as the path along which a unit north pole would tend to move if free to do so.*

Properties of lines of force :

1. Magnetic lines of force are closed curves which start in air from the N-pole and end at the S-pole and then return to the N-pole through the interior of the magnet.
2. The lines of force never cross each other. If they do so, that would mean there are two directions of the magnetic field at the point of intersection, which is impossible.
3. They start *from* and end *on* the surface of the magnet normally.
4. The lines of force have a tendency to contract lengthwise and expand sidewise. This explains attraction between unlike poles and repulsion between like poles.
5. The relative closeness of the lines of force gives a measure of the strength of the magnetic field which is maximum at the poles.

9. *Describe a method for plotting the magnetic field of a bar magnet.*

Plotting magnetic field of a bar magnet. The magnetic field around a magnet can be traced with the help of a magnetic *compass needle.* It consists of a small and light magnetic needle pivoted at the centre of a small circular brass case provided with a glass top, as shown in Fig. 5.11. The north pole of the magnetic needle is generally painted black or red.

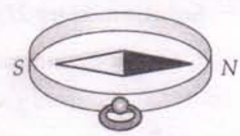


Fig. 5.11 Compass needle.

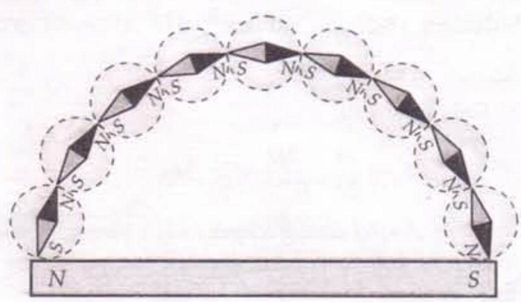


Fig. 5.12 To plot magnetic field of a bar magnet with a compass needle.

To plot the magnetic field of a magnet, the magnet NS is placed on a white paper. The compass needle is placed near its N-pole, as shown in Fig 5.12. The positions of the two ends S and N of the needle are marked by pencil dots on the paper. The needle is now displaced to the new position so that its south pole exactly comes over the mark previously made against the north pole and again the new position of the north pole is marked. The process is repeated till the south pole is reached. The various dots are joined together by a smooth curve which gives a line of force.

Similarly, other lines of force are drawn. A complete pattern of the magnetic field around a bar magnet is shown in Fig. 5.24 on page 5.18.

5.8 MAGNETIC FIELD OF A BAR MAGNET AT AN AXIAL POINT

10. *Derive an expression for the magnetic field intensity at a point on the axis of a bar magnet. What is the direction of the field ?*

Magnetic field of a bar magnet at an axial point (end-on position). Let NS be a bar magnet of length $2l$ and of pole strength q_m. Suppose the magnetic field is to be determined at a point P which lies on the axis of the magnet at a distance r from its centre, as shown in Fig. 5.13.

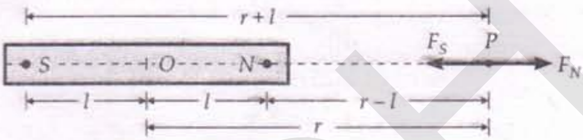


Fig. 5.13 Magnetic field of a bar magnet at an axial point.

Imagine a *unit north pole* placed at point P. Then from Coulomb's law of magnetic forces, the force exerted by the N-pole of strength q_m on unit north pole will be

$$F_N = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{(r-l)^2}, \quad \text{along } \vec{NP}$$

Similarly, the force exerted by S-pole on unit north pole is

$$F_S = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{(r+l)^2}, \quad \text{along } \vec{PS}$$

Therefore, the strength of the magnetic field $\vec{B}$ at point P is

$$B_{\text{axial}} = \text{Force experienced by a unit north-pole at point } P$$

$$= F_N - F_S = \frac{\mu_0 q_m}{4\pi}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$= \frac{\mu_0 q_m}{4\pi} \cdot \frac{4rl}{(r^2-l^2)^2}$$

But $q_m . 2l = m$, is the magnetic dipole moment, so

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2-l^2)^2}$$

For a short bar magnet, $l << r$, therefore, we have

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}, \quad \text{along } \vec{NP} \qquad \text{...(1)}$$

Clearly, *the magnetic field at any axial point of magnetic dipole is in the same direction as that of its magnetic dipole moment i.e.,* from S-pole to N-pole, so we can write

$$\vec{B}_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{m}}{r^3}$$

5.9 MAGNETIC FIELD OF A BAR MAGNET AT AN EQUATORIAL POINT

11. *Derive an expression for the magnetic field intensity at a point on the equatorial line of a bar magnet. What is the direction of this field ?*

Magnetic field of a bar magnet at an equatorial point (broadside-on position). Consider a bar magnet NS of length $2l$ and of pole strength q_m. Suppose the magnetic field is to be determined at a point P lying on the equatorial line of the magnet NS at a distance r from its centre, as shown in Fig. 5.14.

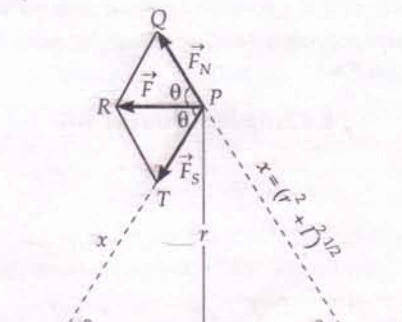


Fig. 5.14 Magnetic field of a bar magnet at an equatorial point.

Imagine a *unit north-pole* placed at point P. Then from Coulomb's law of magnetic forces, the force exerted by the N-pole of the magnet on unit north-pole is

$$F_N = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{x^2}, \quad \text{along } NP$$

Similarly, the force exerted by the S-pole of the magnet on unit north-pole is

$$F_S = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{x^2}, \quad \text{along } PS$$

As the magnitudes of F_N and F_S are equal, so their vertical components get cancelled while the horizontal components add up along PR.

Hence the magnetic field at the equatorial point P is

B_{equa} = Net force on a unit N-pole placed at point P

$$= F_N \cos\theta + F_S \cos\theta$$

$$= 2\, F_N \cos\theta \qquad [\because F_N = F_S]$$

$$= 2 \cdot \frac{\mu_0}{4\pi} \cdot \frac{q_m}{x^2} \cdot \frac{l}{x} \qquad \left[\because \cos\theta = \frac{l}{x}\right]$$

$$\text{or} \quad B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}} \qquad [\because x = (r^2 + l^2)^{1/2}]$$

where $m = q_m \cdot 2l$, is the magnetic dipole moment.

Again for a short magnet, $l << r$, so we have

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}, \text{ along } PR \qquad ...(2)$$

Clearly, *the magnetic field at any equatorial point of a magnetic dipole is in the direction opposite to that of its magnetic dipole moment i.e.,* from N-pole to S-pole. So we can write

$$\vec{B}_{equa} = -\frac{\mu_0 \vec{m}}{4\pi r^3}$$

On comparing equations (1) and (2), we note *that the magnetic field at a point at a certain distance on the axial line of a short magnet is twice of that at the same distance on its equatorial line.*

Examples based on

Magnetic Field of a Bar Magnet

Formulae Used

Magnetic field of a bar magnet of length $2l$ and dipole moment m at a distance r from its centre,

1. $B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2}$ (*on the axial line*)
2. $B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}}$ (*on the equatorial line*)

For a short magnet, $l << r$, so

3. $B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$ (*on the axial line*)
4. $B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$ (*on the equatorial line*)

Units Used

Magnetic field B is in tesla, distances r and l in metre and magnetic moment in JT^{-1} or Am^2.

Example 7. *What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5 cm at a distance of 50 cm from the midpoint ? The magnetic moment of the bar magnet is 0.40 Am^2.* [NCERT]

Solution. Here $m = 0.40$ Am^2, $r = 50$ cm $= 0.50$ m, $2l = 5.0$ cm

Clearly, the magnet is a short magnet ($l << r$).

(*i*) $B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = \frac{10^{-7} \times 0.4}{(0.5)^3} = \mathbf{3.2 \times 10^{-7}}$ **T.**

(*ii*) $B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = \mathbf{6.4 \times 10^{-7}}$ **T.**

Example 8. *A bar magnet of length 10 cm has a pole strength of 10 Am. Calculate the magnetic field at a distance of 0.2 m from its centre at a point on its (i) axial line and (ii) equatorial line.*

Solution. Here $2l = 10$ cm or $l = 5$ cm $= 0.05$ m, $q_m = 10$ Am, $r = 0.2$ m

Magnetic moment,

$$m = q_m \times 2l = 10 \times 0.1 = 1 \text{ Am}^2$$

(*i*) Magnetic field on axial line is

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2} = \frac{10^{-7} \times 2 \times 1 \times 0.2}{(0.2^2 - 0.05^2)^2}$$

$$= \frac{10^{-7} \times 0.4}{(0.0375)^2} \text{ T} = \mathbf{2.84 \times 10^{-5} \text{ T.}}$$

(*ii*) Magnetic field on equatorial line is

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}} = \frac{10^{-7} \times 1}{(0.2^2 + 0.05^2)^{3/2}} \text{ T}$$

$$= \frac{10^{-7}}{(0.0425)^{3/2}} \text{ T} = \frac{10^{-7}}{8.76 \times 10^{-3}} \text{ T}$$

$$= \mathbf{1.14 \times 10^{-5} \text{ T.}}$$

Example 9. *Two small magnets are placed horizontally, perpendicular to the magnetic meridian. Their north poles are at 30 cm east and 20 cm west from a compass needle. If the compass needle remains undeflected, compare the magnetic moments of the magnets.*

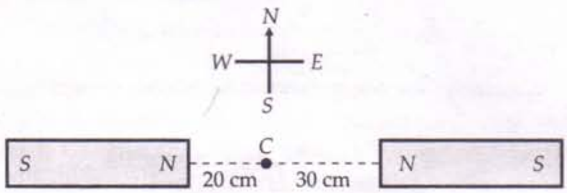


Fig. 5.15

Solution. The compass needle at C lies on the axial line of the two magnets. As it remains undeflected, the fields of the two magnets at C must be equal and opposite.

$$\therefore \quad B_1 = B_2 \quad \text{or} \quad \frac{\mu_0}{4\pi} \cdot \frac{2m_1}{r_1^3} = \frac{\mu_0}{4\pi} \cdot \frac{2m_2}{r_2^3}$$

$$\text{or} \quad \frac{m_1}{m_2} = \left[\frac{r_1}{r_2}\right]^3 = \left[\frac{20}{30}\right]^3 = \frac{8}{27} = \mathbf{8 : 27.}$$

Example 10. *Two short magnets P and Q are placed one over another with their magnetic axes mutually perpendicular to each other. It is found that the resultant field at a point on the prolongation of the magnetic axis of P is inclined at 30° with this axis. Compare the magnetic moments of the two magnets.*

Solution. Let A be any point on the prolongation of the axis of magnet P. Let $\vec{B_1}$ and $\vec{B_2}$ be the fields of the magnets P and Q respectively at the point A. Let $\vec{m_1}$ and $\vec{m_2}$ be the magnetic moments of the two magnets.

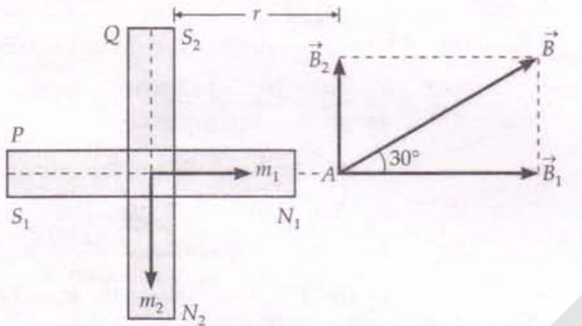


Fig. 5.16

As point A lies on the axial line of P, therefore,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\, m_1}{r^3}$$

The point A lies on the broad-side-on position of Q, therefore,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m_2}{r^3} \qquad \therefore \qquad \frac{B_2}{B_1} = \frac{m_2}{2\, m_1}$$

But the resultant field B_1 is inclined at 30° with B_1, so

$$\frac{B_2}{B_1} = \tan 30° = \frac{1}{\sqrt{3}}$$

Hence $\quad \dfrac{1}{\sqrt{3}} = \dfrac{m_2}{2m_1} \quad$ or $\quad \dfrac{m_1}{m_2} = \dfrac{\sqrt{3}}{2}$.

Example 11. *Two identical magnetic dipoles of magnetic moments 1.0 Am^2 each are placed at a separation of 2 m with their axes perpendicular to each other. What is the resultant magnetic field at a point mid-way between the dipoles ?*

Solution. The situation is shown in Fig. 5.17.

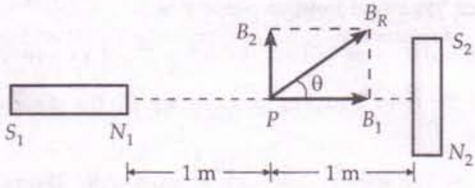


Fig. 5.17

The magnetic fields of the two magnets at the mid-point P are

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = \frac{10^{-7} \times 2 \times 1}{1^3} = 2 \times 10^{-7} \text{ T},$$

in horizontal direction

$$B_2 = \frac{\mu_0}{4\pi} \frac{m}{r^3} = 10^{-7} \text{ T, in vertical direction}$$

$$\therefore \qquad B_R = \sqrt{B_1^2 + B_2^2} = \sqrt{5} \times 10^{-7} \text{ T}$$

If the resultant field B_R makes angle θ with B_1, then

$$\tan\theta = \frac{B_2}{B_1} = \frac{10^{-7}}{2 \times 10^{-7}} = 0.5$$

$$\therefore \qquad \theta = 26.57°$$

Problems For Practice

1. A bar magnet is 0.10 m long and its pole strength is 12 Am. Find the magnitude of the magnetic field at a point on its axis at a distance of 20 cm from it. (Ans. 3.4×10^{-5} T)
2. Calculate the magnetic field due to a bar magnet 2 cm long and having a pole strength of 100 Am at a point 10 cm from each pole. (Ans. 2×10^{-4} T)
3. A bar magnet has a length of 8 cm. The magnetic field at a point at a distance of 3 cm from the centre in the broad-side on position is found to be 4×10^{-6} T. Calculate the pole strength of the magnet. (Ans. 6×10^{-5} Am)
4. The magnetic moment of a current-loop is 2.1×10^{-25} Am^2. Find the magnetic field on the axis of the loop at a distance of 1.0 Å from the loop. (Ans. 4.2×10^{-2} T)
5. If the earth's magnetic field has a magnitude of 3.4×10^{-5} T at the magnetic equator of the earth, what would be its value at the magnetic poles of the earth ? (Ans. 6.8×10^{-5} T)
6. The intensities of magnetic field at two points on the axis of a bar magnet at distances 0.1 m and 0.2 m from its middle point are in the ratio 12.5 : 1. Calculate the distance between the poles of the magnet. (Ans. 0.1 m)
7. Two short magnets a and b of magnetic moments 0.108 Am^2 and 0.192 Am^2 are placed along mutually perpendicular straight lines meeting at a point P. Find the magnitude and direction of magnetic field at point P, if it lies at distances 30 cm and 60 cm respectively from the centres of the two magnets. (Ans. 8.24×10^{-7} T, at 14° with the axis of a)

8. Two short magnets of magnetic moments *a* and *b* of magnetic moments 32 Am^2 and 27 Am^2 are placed on a table, as shown in Fig. 5.18. Find the magnitude and direction of the magnetic field produced

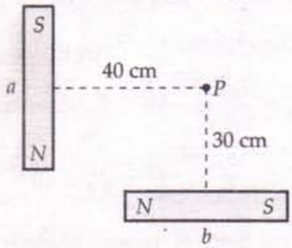


Fig. 5.18

by these magnets at point *P* situated at the equatorial lines of both the magnets at distances 40 cm and 30 cm respectively from the centres of the two magnets. (Ans. 1.12×10^{-4} T, at an angle of 26.57° with the equatorial line of *a*)

HINTS

1. $$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2} = \frac{\mu_0}{4\pi} \cdot \frac{2 \times q_m \times 2l \times r}{(r^2 - l^2)^2}$$
$$= \frac{10^{-7} \times 2 \times 12 \times 0.10 \times 0.20}{(0.2^2 - 0.05^2)^2}$$
$$= \mathbf{3.4 \times 10^{-5} T.}$$

2. $$B_{\text{equa}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{q_m \times 2l}{r^3}$$
$$= \frac{10^7 \times 100 \times 0.02}{(0.10)^3} = \mathbf{2 \times 10^{-4}\ T.}$$

3. Use $$B_{\text{equa}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 - l^2)^{3/2}}$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{q_m \times 2l}{(r^2 - l^2)^{3/2}}.$$

4. The current loop is short magnetic dipole. So
$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = \frac{10^{-7} \times 2 \times 2.1 \times 10^{-25}}{(1.0 \times 10^{-10})^3}$$
$$= \mathbf{4.2 \times 10^{-2} T.}$$

5. At the equator,
$$B = B_{\text{equa}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = 3.4 \times 10^{-5}\text{T}$$
At the poles,
$$B' = B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = 2B$$
$$= \mathbf{6.8 \times 10^{-5}\ T.}$$

7. As shown in Fig. 5.19, the point *P* lies on the axial line of both the magnets *a* and *b*.

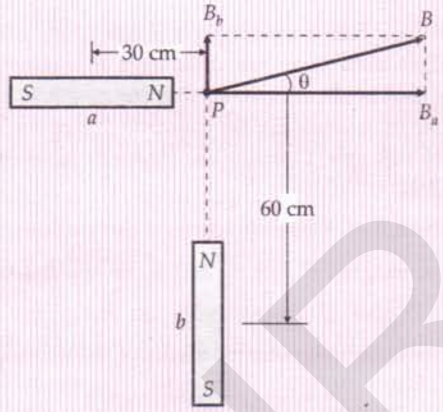


Fig. 5.19

$$\therefore \quad B_a = \frac{\mu_0}{4\pi} \frac{2m_1}{r_1^3} = \frac{10^{-7} \times 2 \times 0.108}{(0.30)^3}$$
$$= 8 \times 10^{-7}\ \text{T} \quad \text{(along the axis of } a)$$
$$B_b = \frac{\mu_0}{4\pi} \cdot \frac{2m_2}{r_2^3} = \frac{10^{-7} \times 2 \times 0.192}{(0.60)^3}$$
$$= 2 \times 10^{-7}\text{T} \quad \text{(along the axis of } b)$$

The resultant field at *P* is
$$B = \sqrt{B_a^2 + B_b^2} = 10^{-7}\sqrt{8^2 + 2^2} = \mathbf{8.24 \times 10^{-7}\ T}$$

If the field *B* makes angle θ with the direction of B_a, then
$$\tan\theta = \frac{B_b}{B_a} = \frac{2}{8} = 0.25 \quad \text{or} \quad \theta = \mathbf{14°}.$$

8. As shown in Fig. 5.20, the point *P* lies on the equatorial line of both the magnets *a* and *b*.

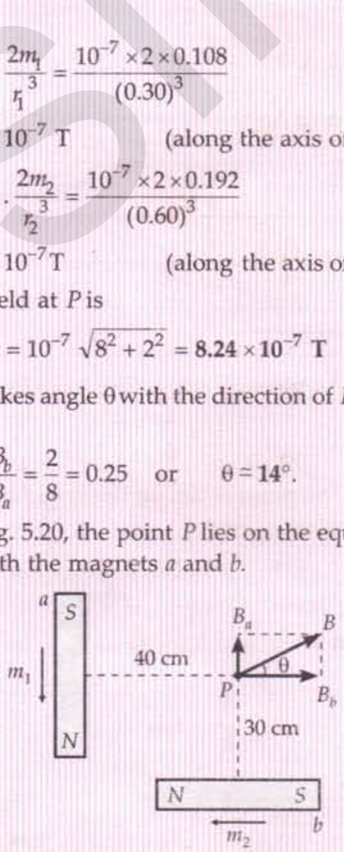


Fig. 5.20

$$\therefore \quad B_a = \frac{\mu_0}{4\pi} \cdot \frac{m_1}{r_1^3} = \frac{10^{-7} \times 32}{(0.40)^3} = 0.5 \times 10^{-4}\text{T} \quad \text{(antiparallel to } m_1)$$
$$B_b = \frac{\mu_0}{4\pi} \cdot \frac{m_2}{r_2^3} = \frac{10^{-7} \times 27}{(0.30)^3} = 10^{-4}\text{T} \quad \text{(antiparallel to } m_2)$$

The two fields are perpendicular to each other. So the resultant field at point *P* is
$$B = \sqrt{B_a^2 + B_b^2} = 10^{-4}\sqrt{(0.5)^2 + 1^2} = \mathbf{1.12 \times 10^{-4}\ T}$$

If the field *B* makes an angle θ with the direction of B_b, then
$$\tan\theta = \frac{B_a}{B_b} = \frac{0.5 \times 10^{-4}}{10^{-4}} = 0.5 \quad \text{or} \quad \theta = \mathbf{26.57°}$$

5.10 TORQUE ON A MAGNETIC DIPOLE IN A MAGNETIC FIELD

12. *Derive an expression for the torque on a magnetic dipole placed in a uniform magnetic field. Hence define magnetic dipole moment.*

Torque on a magnetic dipole in a uniform magnetic field. Consider a bar magnet *NS* of length $2l$ placed in a uniform magnetic field $\vec{B}$. Let q_m be the pole strength of its each pole. Let the magnetic axis of the bar magnet make an angle θ with the field $\vec{B}$, as shown in Fig. 5.21(*a*).

Force on N-pole $= q_m B$; along $\vec{B}$

Force on S-pole $= q_m B$, opposite to $\vec{B}$

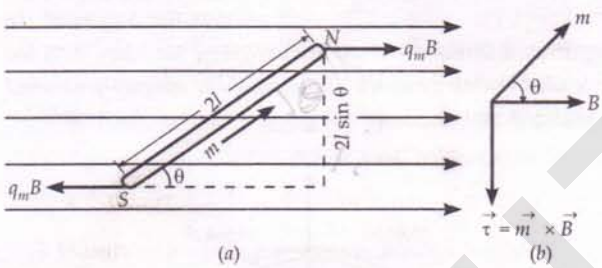


Fig. 5.21 (*a*) Torque on a bar magnet in a magnetic field.
(*b*) Relation between the directions of $\vec{\tau}$, $\vec{m}$, $\vec{B}$.

The forces on the two poles are equal and opposite. They form a couple. Moment of couple or torque is given by

$$\tau = \text{Force} \times \text{perpendicular distance}$$

$$= q_m B \times 2l \sin\theta = (q_m \times 2l)\, B \sin\theta$$

or $$\tau = mB \sin\theta \qquad ...(1)$$

where $m = q_m \times 2l$, is the magnetic dipole moment of the bar magnet. In vector notation,

$$\vec{\tau} = \vec{m} \times \vec{B} \qquad ...(2)$$

The direction of the torque $\vec{\tau}$ is given by the right hand screw rule as indicated in Fig. 5.21(*b*). The effect of the torque $\vec{\tau}$ is to make the magnet align itself parallel to the field $\vec{B}$. That is why a freely suspended magnet aligns itself in the north-south direction because the earth has its own magnetic field which exerts a torque on the magnet tending it to align along the field.

Special Cases

1. When the magnet lies along the direction of the magnetic field,
$$\theta = 0°, \quad \sin\theta = 0, \quad \tau = 0,$$
Thus the torque is *minimum*.
2. When the magnet lies perpendicular to the direction of the field,
$$\theta = 90°, \quad \sin\theta = 1, \quad \tau = mB$$
Thus the torque is *maximum*.
$$\tau_{max} = mB$$

Definition of magnetic dipole moment. If in Eq. (1), $B = 1$, $\theta = 90°$, then

$$\tau = m$$

Hence *the magnetic dipole moment may be defined as the torque acting on a magnetic dipole placed perpendicular to a uniform magnetic field of unit strength.*

SI unit of magnetic moment. As

$$m = \frac{\tau}{B \sin\theta}$$

$$\therefore \quad \text{SI units of } m = \frac{1\,\text{Nm}}{1\,\text{T}.1}$$

$$= \mathbf{NmT^{-1} \text{ or } JT^{-1} \text{ or } Am^2}.$$

5.11 POTENTIAL ENERGY OF A MAGNETIC DIPOLE IN A MAGNETIC FIELD

13. *Derive an expression for the potential energy of a dipole placed in a uniform magnetic field at an angle θ with it. When will the magnetic dipole be in the positions of stable and unstable equilibrium ?*

Potential energy of a magnetic dipole. As shown in Fig. 5.21(*a*), when a magnetic dipole is placed in a uniform magnetic field $\vec{B}$ at angle θ with it, it experiences a torque

$$\tau = mB \sin\theta$$

This torque tends to align the dipole in the direction of $\vec{B}$.

If the dipole is rotated against the action of this torque, work has to be done. This work is stored as *potential energy* of the dipole.

The work done in turning the dipole through a small angle $d\theta$ is

$$dW = \tau\, d\theta = mB \sin\theta\, d\theta$$

If the dipole is rotated from an initial position $\theta = \theta_1$ to the final position $\theta = \theta_2$, then the total work done will be

$$W = \int dW = \int_{\theta_1}^{\theta_2} mB \sin\theta \, d\theta = mB[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= -mB(\cos\theta_2 - \cos\theta_1)$$

This work done is stored as the potential energy U of the dipole.

$$\therefore \quad U = -mB(\cos\theta_2 - \cos\theta_1)$$

The potential energy of the dipole is zero when $\vec{m} \perp \vec{B}$. So potential energy of the dipole in any orientation θ can be obtained by putting $\theta_1 = 90°$ and $\theta_2 = \theta$ in the above equation.

$$\therefore \quad U = -mB(\cos\theta - \cos 90°)$$

or $$U = -mB\cos\theta = -\vec{m} \cdot \vec{B}$$

Special Cases

1. When $\theta = 0°$, $U = -mB\cos 0° = -mB$
   Thus the potential energy of a dipole is *minimum* when $\vec{m}$ is *parallel* to $\vec{B}$. In this state, the magnetic dipole is in ***stable equilibrium***.
2. When $\theta = 90°$, $U = -mB\cos 90° = 0$.
3. When $\theta = 180°$, $U = -mB\cos 180° = +mB$.

Thus the potential energy of a dipole is *maximum* when $\vec{m}$ is *antiparallel* to $\vec{B}$. In this state, the magnetic dipole is in ***unstable equilibrium***.

5.12 CURRENT LOOP AS A MAGNETIC DIPOLE

14. *Show that a current carrying loop behaves as a magnetic dipole. Hence write an expression for its magnetic dipole moment.*

Current loop as a magnetic dipole. We know that the magnetic field produced at a large distance r from the centre of a circular loop (of radius a) along its axis is given by

$$B = \frac{\mu_0 I a^2}{2 r^3}$$

or $$B = \frac{\mu_0}{4\pi} \cdot \frac{2IA}{r^3} \quad \text{...(1)}$$

where I is the current in the loop and $A = \pi a^2$ is its area. On the other hand, the electric field of an electric dipole at an axial point lying far away from it is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} \quad \text{...(2)}$$

where p is the electric dipole moment of the electric dipole.

On comparing equations (1) and (2), we note that both B and E have same distance dependence $\left(\frac{1}{r^3}\right)$. Moreover, they have same direction at any far away point, not just on the axis. This suggests that a circular current loop behaves as a magnetic dipole of magnetic moment,

$$m = IA$$

In vector notation,

$$\vec{m} = I\vec{A} = IA\hat{n}$$

This result is valid for planar current loop of any shape. *Thus the magnetic dipole moment of any current loop is equal to the product of the current and its loop area.* Its direction is defined to be normal to the plane of the loop in the sense given by right hand thumb rule.

Right hand thumb rule. *If we curl the fingers of the right hand in the direction of current in the loop, then the extended thumb gives the direction of the magnetic moment associated with the loop.*

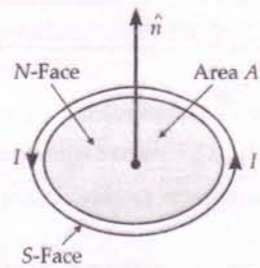


Fig. 5.22 Current loop as a magnetic dipole.

It follows from the above rule that the upper face of the current loop shown in Fig. 5.22. has N-polarity and the lower face has S-polarity. Thus *a current loop behaves like a magnetic dipole.*

If a current carrying coil consists of N turns, then

$$m = NIA$$

The factor NI is called *amperes turns* of current loop. So,

Magnetic dipole moment of current loop
= Ampere turns × loop area

Clearly, dimensions of magnetic moment

$$= [A][L^2] = [AL^2]$$

SI unit of magnetic dipole moment is Am^2. *It is defined as the magnetic moment associated with one turn loop of area one square metre when a current of one ampere flows through it.*

Table 5.1 Analogy between electric and magnetic dipoles

Physical quantity	Electrostatics	Magnetism
Free space constant	$\frac{1}{\varepsilon_0}$	μ_0
Dipole moment	$\vec{p}$	$\vec{m}$
Axial field	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2\vec{p}}{r^3}$	$\frac{\mu_0}{4\pi} \cdot \frac{2\vec{m}}{r^3}$
Equatorial field	$-\frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}}{r^3}$	$-\frac{\mu_0}{4\pi} \cdot \frac{\vec{m}}{r^3}$
Torque in external field	$\vec{p} \times \vec{E}$	$\vec{m} \times \vec{B}$
P.E. in external field	$-\vec{p} \cdot \vec{E}$	$-\vec{m} \cdot \vec{B}$

5.13 MAGNETIC DIPOLE MOMENT OF A REVOLVING ELECTRON

15. *Derive an expression for the magnetic dipole moment of an electron revolving around a nucleus. Define Bohr magneton and find its value.*

Magnetic dipole moment of a revolving electron. According to Bohr model of hydrogen-like atoms, negatively charged electron revolves around the positively charged nucleus. This uniform circular motion of the electron is equivalent to a current loop which possesses a magnetic dipole moment = IA. As shown in Fig. 5.23, consider an electron revolving anticlockwise around a nucleus in an orbit of radius r with speed v and time period T.

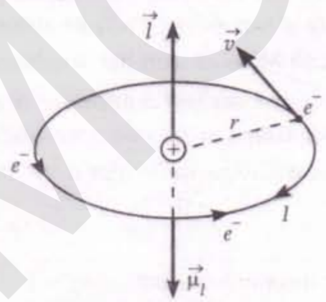


Fig. 5.23 **Orbital magnetic moment of a revolving electron.**

Equivalent current,

$$I = \frac{\text{Charge}}{\text{Time}} = \frac{e}{T} = \frac{e}{2\pi r / v} = \frac{ev}{2\pi r}$$

Area of the current loop, $A = \pi r^2$

Therefore, the *orbital magnetic moment* (magnetic moment due to orbital motion) of the electron is

$$\mu_l = IA = \frac{ev}{2\pi r} \cdot \pi r^2$$

or $$\mu_l = \frac{evr}{2} \qquad ...(1)$$

As the negatively charged electron is revolving anticlockwise, the associated current flows clockwise. According to right hand thumb rule, the direction of the magnetic dipole moment of the revolving electron will be perpendicular to the plane of its orbit and in the downward direction, as shown in Fig. 5.23

Also, the angular momentum of the electron due to its orbital motion is

$$l = m_e v r \qquad ...(2)$$

The direction of $\vec{l}$ is normal to the plane of the electron orbit and in the upward direction, as shown in Fig. 5.23.

Dividing equation (1) by (2), we get

$$\frac{\mu_l}{l} = \frac{evr/2}{m_e vr} = \frac{e}{2m_e}$$

The above ratio is a constant called ***gyromagnetic ratio***. Its value is 8.8×10^{10} C kg^{-1}. So

$$\mu_l = \frac{e}{2m_e} l$$

Vectorially,

$$\vec{\mu}_l = -\frac{e}{2m_e} \vec{l}$$

The negative sign shows that the direction of $\vec{l}$ is opposite to that of $\vec{\mu}_l$. According to *Bohr's quantisation condition*, the angular momentum of an electron in any permissible orbit is integral multiple of $h/2\pi$, where h is Planck's constant, *i.e.*,

$$l = \frac{nh}{2\pi}, \quad \text{where } n = 1, 2, 3, ...$$

$$\therefore \qquad \mu_l = n\left(\frac{eh}{4\pi m_e}\right)$$

This equation gives orbital magnetic moment of an electron revolving in nth orbit.

Bohr magneton. *It is defined as the magnetic moment associated with an electron due to its orbital motion in the first orbit of hydrogen atom.* It is the minimum value of μ_l which can be obtained by putting $n = 1$ in the above equation. Thus Bohr magneton is given by

$$\mu_B = (\mu_l)_{\min} = \frac{eh}{4\pi m_e}$$

Putting the values of various constants, we get

$$\mu_B = \frac{1.6 \times 10^{-19}\ \text{C} \times 6.63 \times 10^{-34}\ \text{Js}}{4 \times 3.14 \times 9.11 \times 10^{-31}\ \text{kg}}$$

$$= \mathbf{9.27 \times 10^{-24}\ Am^2}.$$

Besides the orbital angular momentum $\vec{l}$, an electron has spin angular momentum $\vec{S}$ due to its spinning motion. *The magnetic moment possessed by an electron due to its spinning motion is called* ***intrinsic magnetic moment*** *or* ***spin magnetic moment***. It is given by

$$\vec{\mu}_s = -\frac{e}{m_e}\vec{S}$$

The total magnetic moment of the electron is the vector sum of these two momenta. It is given by

$$\vec{\mu} = \vec{\mu}_l + \vec{\mu}_s = -\frac{e}{2m_e}(\vec{l} + 2\vec{S})$$

Examples based on

Torque and Potential Energy of a Dipole, and Magnetic Moment of a Current Loop

Formulae Used

1. Torque, $\tau = mB \sin\theta$ or $\vec{\tau} = \vec{m} \times \vec{B}$
2. Work done in turning the dipole or P.E. of a dipole,
   $W = U = -mB(\cos\theta_2 - \cos\theta_1)$
3. If initially the dipole is perpendicular to the field,
   $U = -mB\cos\theta$
   (*i*) When $\vec{m}$ is parallel to $\vec{B}$, $\theta = 0°$, $U = -mB$
   Potential energy of the dipole is minimum. It is in a state of stable equilibrium.
   (*ii*) When $\vec{m}$ is *perpendicular* to $\vec{B}$, $\theta = 90°$, $U = 0$.
   (*iii*) When $\vec{m}$ is *antiparallel* to $\vec{B}$, $\theta = 180°$, $U = +mB$
   Potential energy of the dipole is maximum. It is in a state of unstable equilibrium.
4. Magnetic moment of a current loop, $m = NIA$
5. Orbital magnetic moment of an electron in *n*th orbit,
   $$\mu_l = \frac{evr}{2} = \frac{e}{2m_e} l = n\left(\frac{eh}{4\pi m_e}\right)$$
6. Bohr magneton is the magnetic moment of an electron in first ($n = 1$) orbit.
   $$\mu_B = (\mu_l)_{min} = \frac{eh}{4\pi m_e}.$$

Units Used

Torque τ is in Nm, magnetic moment m in JT^{-1} or Am^2, field B in tesla, potential energy U in joule.

Example 12. *A magnetised needle of magnetic moment $4.8 \times 10^{-2}\ JT^{-1}$ is placed at 30° with the direction of uniform magnetic field of magnitude 3×10^{-2} T. What is the torque acting on the needle ?* **[CBSE D 01C]**

Solution. Here $m = 4.8 \times 10^{-2}\ JT^{-1}$, $\theta = 30°$, $B = 3 \times 10^{-2}$ T

∴ Torque, $\tau = mB\sin\theta$

$= 4.8 \times 10^{-2} \times 3 \times 10^{-2} \times \sin 30°$

$= \mathbf{7.2 \times 10^{-4}\ J}.$

Example 13. *A short bar magnet placed with its axis at 30° to a uniform magnetic field of 0.2 T experiences a torque of 0.06 Nm. (i) Calculate the magnetic moment of the magnet. (ii) Find out what orientation of the magnet corresponds to its stable equilibrium in the magnetic field.* **[CBSE OD 02]**

Solution. (*i*) Here $B = 0.2$ T, $\theta = 30°$, $\tau = 0.06$ Nm

Magnetic moment,

$$m = \frac{\tau}{B\sin\theta} = \frac{0.06}{0.2\sin 30°} = \frac{0.06}{0.2 \times 0.5} = \mathbf{0.6\ Am^2}.$$

(*ii*) The P.E. of a magnetic dipole in a uniform magnetic field is

$$U = -mB\cos\theta$$

In stable equilibrium, the P.E. is minimum. So

$$\cos\theta = 1 \quad \text{or} \quad \theta = 0°$$

Hence the bar magnet will be in stable equilibrium when its magnetic moment $\vec{m}$ is parallel to the magnetic field $\vec{B}$.

Example 14. *In an iron bar (5 cm × 1 cm × 1 cm) the magnetic moment of an atom is $1.8 \times 10^{-23}\ Am^2$. (i) What will be magnetic moment of the bar in the state of magnetic saturation ? (ii) What torque will have to be applied to keep the bar perpendicular to an external magnetic field of 15,000 gauss ? Density of iron = 7.8 g cm^{-3}, its atomic mass = 56.*

Solution. (*i*) Mass of iron bar = volume × density

$= 5\ cm^3 \times 7.8\ g\ cm^{-3} = 39$ g

Number of atoms in 56 g of iron $= 6.02 \times 10^{23}$

∴ Number of atoms in 39 g of iron

$$= \frac{6.02 \times 10^{23} \times 39}{56} = 4.19 \times 10^{23}$$

Magnetic moment of each atom

$= 1.8 \times 10^{-23}\ Am^2$

Magnetic moment of the iron bar in the state of magnetic saturation is

$$m = 1.8 \times 10^{-23} \times 4.19 \times 10^{23} = \mathbf{7.54\ Am^2}.$$

(*ii*) Here $\theta = 90°$, $B = 15{,}000\ G = 15000 \times 10^{-4}$ T

∴ Required torque, $\tau = mB\sin\theta$

$= 7.54 \times 15000 \times 10^{-4} \times \sin 90° = \mathbf{11.3\ Nm.}$

Example 15. *A planar loop of irregular shape encloses an area of* $7.5 \times 10^{-4}\, m^2$ *and carries a current of 12 A. The sense of flow of current appears to be clockwise to an observer. What is the magnitude and direction of the magnetic moment vector associated with the current loop ?* [NCERT]

Solution. Here $A = 7.5 \times 10^{-4}\ \text{m}^2$, $I = 12$ A

Magnetic moment associated with the loop is

$$m = IA = 12 \times 7.5 \times 10^{-4}$$

$$= \mathbf{9.0 \times 10^{-3}\ JT^{-1}}$$

Applying right hand rule, the direction of magnetic moment is along the normal to the plane of the loop away from the observer.

Example 16. *A current of 5 A is flowing through a 10 turn circular coil of radius 7 cm. The coil lies in the x-y plane. What is the magnitude and direction of the magnetic dipole moment associated with it ?*

If this coil were to be placed in a uniform external magnetic field directed along the x-axis, in which plane would the coil lie, when in equilibrium ? (Take $\pi = 22/7$*)* [CBSE Sample Paper 03]

Solution. Magnetic dipole moment,

$$m = NIA = NI \times \pi r^2$$

$$= 10 \times 5 \times \frac{22}{7} \times \left(\frac{7}{100}\right)^2$$

$$= \mathbf{0.77\ Am^2}$$

The direction of magnetic dipole moment is perpendicular to the plane of the coil. Hence it is along z-axis.

Torque on the current loop of magnetic moment m is

$$\tau = mB \sin \alpha$$

where α is angle between $\vec{m}$ and $\vec{B}$. For stable equilibrium torque is zero, so $\alpha = 0°$. For this $\vec{B}$ should be perpendicular to the plane of the coil. Hence the coil will lie in y-z plane in the condition of stable equilibrium.

Example 17. *A bar magnet with poles 25 cm apart and of pole strength 14.4 Am rests with its centre on a frictionless pivot. It is held in equilibrium at 60° to a uniform magnetic field of induction 0.25 T by applying a force F, at right angles to its axis, 12 cm from its pivot. Calculate F. What will happen if the force F is removed ?* [IIT]

Solution. Here $m = q_m \times 2l = 14.4 \times 0.25 = 3.6\ \text{Am}^2$

$\theta = 60°$, $B = 0.25$ T, $r = 12$ cm $= 0.12$ m

Torque, $\tau = Fr = mB \sin\theta$

$$\therefore \quad F = \frac{mB \sin\theta}{r} = \frac{3.6 \times 0.25 \times \sin 60°}{0.12}$$

$$= \frac{3.6 \times 0.25 \times 0.866}{0.12} = \mathbf{6.5\ N}$$

When the force F is removed, the magnet aligns itself in the direction of field B.

Example 18. *An electron in an atom revolves around the nucleus in an orbit of radius 0.5 Å. Calculate the equivalent magnetic moment if the frequency of revolution of the electron is* 10^{10} *MHz.* [IIT 88, CBSE D 98]

Solution. The electron revolving around the nucleus in a circular orbit is equivalent to a current loop. Its magnetic moment is $m = IA = e\nu \times \pi r^2$

Here $e = 1.6 \times 10^{-19}$ C, $\nu = 10^{10}$ MHz $= 10^{16}$ Hz, $r = 0.5$ Å $= 0.5 \times 10^{-10}$ m

$$\therefore \quad m = 1.6 \times 10^{-19} \times 10^{16} \times 3.14 \times (0.5 \times 10^{-10})^2$$

$$= \mathbf{1.256 \times 10^{-23}\ Am^2}.$$

Example 19. *An electron moves around the nucleus in a hydrogen atom of radius 0.51 Å, with a velocity of* 2×10^6 *m/s. Calculate the following :*

(*i*) *the equivalent current due to orbital motion of electron*

(*ii*) *the magnetic field produced at the centre of the nucleus*

(*iii*) *the magnetic moment associated with the electron.* [CBSE OD 08]

Solution. Here $r = 0.51 \times 10^{-10}$ m, $v = 2 \times 10^5\ \text{ms}^{-1}$

(*i*) $I = \dfrac{e}{T} = \dfrac{ev}{2\pi r} = \dfrac{1.6 \times 10^{-19} \times 2 \times 10^5}{2\pi \times 0.51 \times 10^{-10}} = \mathbf{10^{-4}\ A.}$

(*ii*) $B = \dfrac{\mu_0 I}{2r} = \dfrac{4\pi \times 10^{-7} \times 10^{-4}}{2 \times 0.51 \times 10^{-10}} = \mathbf{1.23\ T.}$

(*iii*) $m = IA = \dfrac{ev}{2\pi r} \times \pi r^2 = \dfrac{evr}{2}$

$$= \frac{1.6 \times 10^{-19} \times 2 \times 10^5 \times 0.51 \times 10^{-10}}{2}$$

$$= \mathbf{8.16 \times 10^{-25}\ Am^2}.$$

Example 20. *Two magnets of magnetic moments m and* $\sqrt{3}$ *m are joined to form a cross (+). The combination is suspended freely in a uniform magnetic field. In equilibrium position, the magnet of magnetic moment m makes an angle θ with the field. Find θ.*

Solution. When the magnet of moment m makes angle θ with the field B, the other magnet of moment $\sqrt{3}\, m$ will make angle $(90° - \theta)$ with the field B. In the equilibrium position,

Torque experienced by first magnet

= Torque experienced by second magnet

or $\quad mB \sin\theta = \sqrt{3}\, mB \sin(90° - \theta)$

or $\sin\theta = \sqrt{3}\cos\theta$

or $\frac{\sin\theta}{\cos\theta} = \sqrt{3}$ or $\tan\theta = \sqrt{3}$

$\therefore$ $\theta = \mathbf{60°}$.

Example 21. *A bar magnet having a magnetic moment of 1.0×10^4 JT^{-1} is free to rotate in a horizontal plane. A horizontal magnetic field of 4×10^{-5} T exists in space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.*

Solution. Here $m = 1.0 \times 10^4$ JT^{-1}, $B = 4 \times 10^{-5}$ T, $\theta_1 = 0°$, $\theta_2 = 60°$

Work done,

$$W = -mB(\cos\theta_2 - \cos\theta_1)$$
$$= -1.0 \times 10^4 \times 4 \times 10^{-5}(\cos 60° - \cos 0°)$$
$$= 1.0 \times 10^4 \times 4 \times 10^{-5} \times \frac{1}{2} = \mathbf{0.2\ J.}$$

Example 22. *A current of 7.0 A is flowing in a plane circular coil of radius 1.0 cm having 100 turns. The coil is placed in a uniform magnetic field of 0.2 Wbm^{-2}. If the coil is free to rotate, what orientations would correspond to its (i) stable equilibrium and (ii) unstable equilibrium ? Calculate the potential energy of the coil in these cases.* [CBSE OD 92]

Solution. Here $N = 100$, $A = 7.0$ A, $r = 1.0$ cm $= 1.0 \times 10^{-2}$ m, $B = 0.2$ Wb m^{-2}

Magnetic moment associated with the coil is

$$m = NIA = NI \times \pi r^2$$
$$= 100 \times 7.0 \times \frac{22}{7} \times (1.0 \times 10^{-2})^2 = 0.22\ Am^2$$

(*i*) The stable equilibrium corresponds to $\vec{m}$ parallel to $\vec{B}$. The potential energy is then minimum.

$$U_{min} = -mB\cos 0° = -0.22 \times 0.2 \times 1 = \mathbf{-0.044\ J.}$$

(*ii*) The unstable equilibrium corresponds to $\vec{m}$ anti-parallel to $\vec{B}$. The potential energy is then maximum.

$$U_{max} = -mB\cos 180° = -0.22 \times 0.2 \times (-1)$$
$$= \mathbf{+0.044\ J.}$$

Example 23. *A short bar magnet placed with its axis at 30° experiences a torque of 0.016 Nm in an external field of 800 G. (a) What is the magnetic moment of the magnet ? (b) What is the work done by an external force in moving it from its most stable to most unstable position ? (c) What is the work done by the force due to the external magnetic field in the process mentioned in part (b) ? (d) The bar magnet is replaced by a solenoid of cross-sectional area 2×10^{-4} m^2 and 1000 turns, but the same magnetic moment. Determine the current flowing through the solenoid.* [NCERT]

Solution. (*a*) Here $\theta = 30°$, $B = 800$ G $= 800 \times 10^{-4}$ T, $\tau = 0.016$ Nm

Magnetic moment,

$$m = \frac{\tau}{B\sin\theta}$$
$$= \frac{0.016}{800 \times 10^{-4} \times \sin 30°} = \mathbf{0.40\ Am^2}.$$

(*b*) For most stable position, $\theta = 0°$ and for most unstable position $\theta = 180°$. So the required work done by the external force,

$$W = -mB(\cos 180° - \cos 0°) = 2mB$$
$$= 2 \times 0.40 \times 800 \times 10^{-4} = \mathbf{0.064\ J.}$$

(*c*) Here the displacement and the torque due to the magnetic field are in opposition. So the work done by the magnetic field due to the external magnetic field is

$$W_B = \mathbf{-0.064\ J.}$$

(*d*) Here $A = 2 \times 10^{-4}$ m^2, $N = 1000$

Magnetic moment of solenoid,

$$m_s = m = 0.40\ Am^2$$

But $m_s = NIA$

$\therefore$ Current, $I = \frac{m_s}{NA} = \frac{0.40}{1000 \times 2 \times 10^{-4}} = \mathbf{2\ A.}$

Problems For Practice

1. A short bar magnet of magnetic moment 0.9 JT^{-1} is placed with its axis at 30° to a uniform magnetic field. It experiences a torque of 0.063 J. (*i*) Calculate the magnitude of the magnetic field. (*ii*) In which orientation will the bar magnet be in stable equilibrium in the magnetic field ? **[CBSE F 12]**

   (Ans. 0.14 T, $\vec{m} \parallel \vec{B}$ for stable equilibrium)

2. A circular coil of 300 turns and diameter 14 cm carries a current of 15 A. What is the magnitude of magnetic moment associated with the loop ?

   [Haryana 01] (Ans. 69.3 JT^{-1})

3. Calculate the magnitude of the torque required to hold a bar magnet of magnetic moment 200 Am^2 along a direction making an angle of 30° with the direction of a uniform magnetic field of 0.36 G.

   (Ans. 3.6×10^{-3} Nm)

4. Calculate the torque acting on a magnet of length 20 cm and of pole strength 2×10^{-5} Am, placed in earth's magnetic field of flux density 2×10^{-5} T, when (*i*) magnet is parallel to the field (*ii*) magnet is perpendicular to the field.

   [Ans. (*i*) Zero (*ii*) 0.8×10^{-10} Nm]

5. The magnetic dipole moment of the earth is 6.4×10^{21} Am^2. If we consider it to be due to a current loop wound round the magnetic equator of the earth, then what should be the magnitude of the current ? Assume the earth to be sphere of radius 6400 km. (Ans. 5×10^7 A)

6. A straight solenoid of length 50 cm has 1000 turns and a mean cross-sectional area of 2×10^{-4} m^2. It is placed with its axis at 30°, with a uniform magnetic field of 0.32 T. Find the torque acting on the solenoid when a current of 2 A is passed through it. (Ans. 0.064 Nm)

7. A current of 3 A flows through a plane circular coil of radius 4 cm and having 20 number of turns. The coil has been placed in a uniform magnetic field of 0.5 T. Find (*i*) dipole moment of the coil (*ii*) potential energy of the dipole. (Ans. 0.3 Am^2, – 0.15 J)

8. A bar magnet placed in a uniform magnetic field of strength 0.3 T with its axis at 30° to the field, experiences a torque of 0.06 Nm. What is the magnetic moment of the bar magnet ? **[ISCE 98]** (Ans. 0.4 Am^2)

9. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.16 T experiences a torque of magnitude 0.032 J.
   (*a*) Estimate the magnetic moment of the magnet.
   (*b*) If the bar were free to rotate, which orientations would correspond to its
   (*i*) stable, and (*ii*) unstable equilibrium ?
   What is its potential energy in the field for cases (*i*) and (*ii*) ?
   [Ans. (*a*) 0.40 JT^{-1}, (*b*) (*i*) – 0.064 J (*ii*) + 0.064 J]

10. Calculate the work done in rotating a magnet of magnetic moment 3.0 J T^{-1} through an angle of 60° from its position along a magnetic field of strength 0.34×10^{-4} T. (Ans. 5.1×10^{-5} J)

11. A bar magnet of magnetic moment 2.5 Am^2 is free to rotate about a vertical axis through its centre. The magnet is released from rest from the east-west direction. Find the kinetic energy of the magnet as it aligns itself in the north-south direction. The horizontal component of earth's magnetic field is 0.3 G. (Ans. 75 μJ)

HINTS

1. Here $m = 0.9\ \text{J T}^{-1}$, $\theta = 30°$, $\tau = 0.063$ J

   (*i*) $B = \dfrac{\tau}{m \sin\theta} = \dfrac{0.063}{0.9 \times \sin 30°} = \mathbf{0.14\ T}$

   (*ii*) When $\theta = 0°$, $U = -mB\cos 0° = -mB$

   The P.E. of the magnet is minimum. Hence the bar magnet will be in stable equilibrium when $\vec{m}$ is parallel to $\vec{B}$.

2. Here $N = 300$, $I = 15$ A, $r = 7\text{ cm} = 7 \times 10^{-2}$ m

   $\therefore\ m = NIA = NI \times \pi r^2 = 300 \times 15 \times 3.14 \times (7 \times 10^{-2})^2$
   $= \mathbf{69.3\ J\,T^{-1}}$.

5. Use $m = IA = I \times \pi r^2$.

6. $\tau = mB\sin\theta = NIA\ B\sin\theta$
   $= 1000 \times 2 \times 2 \times 10^{-4} \times 0.32 \times \sin 30°$
   $= \mathbf{0.064\ Nm.}$

7. (*i*) $m = NIA = NI \times \pi r^2 = 20 \times 3 \times 3.14 \times (0.04)^2$
   $= \mathbf{0.3\ Am^2}$

   (*ii*) In equilibrium position,
   $U = -mB = -0.3 \times 0.5 = \mathbf{-0.15\ J.}$

8. $m = \dfrac{\tau}{B\sin\theta} = \dfrac{0.06}{0.3\sin 30°} = \mathbf{0.4\ Am^2}$.

9. (*a*) Here $\theta = 30°$, $B = 0.16$ T, $\tau = 0.032$ J

   Magnetic moment,
   $$m = \frac{\tau}{B\sin\theta} = \frac{0.032}{0.16 \times \sin 30°} = \mathbf{0.40\ J\,T^{-1}}.$$

   (*b*) Potential energy of the dipole in a magnetic field $\vec{B}$ is given by
   $$U = -\vec{m}\cdot\vec{B} = -mB\sin\theta$$

   (*i*) The bar will be in stable equilibrium when its magnetic moment $\vec{m}$ is parallel to $\vec{B}$ ($\theta = 0°$). Its potential energy is then minimum and is given by
   $$U_{min} = -mB\cos 0° = -mB$$
   $$= -0.40 \times 0.16 = \mathbf{-0.064\ J.}$$

   (*ii*) The bar will be in unstable equilibrium when $\vec{m}$ is antiparallel to $\vec{B}$ ($\theta = 180°$). Its potential energy is then maximum and is given by
   $U_{max} = -mB\cos 180° = +mB = \mathbf{+0.064\ J}$

10. $W = -mB(\cos\theta_2 - \cos\theta_1)$
   $= -3.0 \times 0.34 \times 10^{-4}(\cos 60° - \cos 0°)$
   $= \mathbf{5.1 \times 10^{-5}\ J.}$

11. Here $\theta_1 = 90°$, $\theta_2 = 0°$, $m = 2.5$ Am^2,
   $B = 0.3\text{ G} = 0.3 \times 10^{-4}$ T

   Kinetic energy = Loss in P.E.
   $= U_i - U_f = -mB\cos 90° + mB\cos 0°$
   $= 2.5 \times 0.3 \times 10^{-4}\ \text{J} = \mathbf{75\ \mu J.}$

5.14 BAR MAGNET AS AN EQUIVALENT SOLENOID

16. *State some similarities between a current carrying solenoid and a bar magnet.*

Similarities between a current carrying solenoid and a bar magnet. When a current is passed through a solenoid, it behaves like a bar magnet. Some observations of similar behaviour are as follows :

1. A current carrying solenoid suspended freely always comes to rest in north-south direction.
2. Two current-carrying solenoids exhibit mutual attraction and repulsion when brought closer to one another. This shows that their end faces act as N-and S-poles like that of a bar magnet.
3. Figure 5.24 shows the lines of force of a bar magnet while Fig. 5.25 shows the lines of force of a finite solenoid. The two patterns have a striking resemblance.

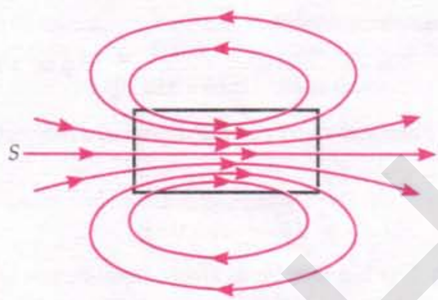


Fig. 5.24 Field lines of a bar magnet.

If we move a small compass needle in the neighbourhood of the bar magnet and the current carrying finite solenoid, we shall find that deflections of the needle are similar in the two cases. This again supports the similarity between the two fields.

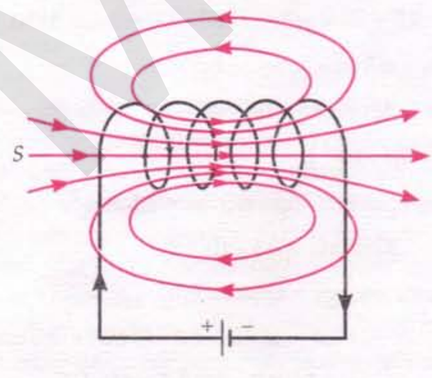


Fig. 5.25 Field lines of a current carrying finite solenoid.

4. The magnetic fields of both the bar magnet and current carrying solenoid at any far away axial point are given by the same expression :

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

Thus a bar magnet and a solenoid produce similar magnetic fields.

17. *Explain how is a current carrying solenoid equivalent to a bar magnet.*

A solenoid as an equivalent bar magnet. A solenoid can be regarded as a combination of circular loops placed side by side, as shown in Fig. 5.26(*a*). Each turn of the solenoid can be regarded as a small magnetic dipole of dipole moment *IA*. Then the solenoid becomes an arrangement of small magnetic dipoles placed in line with each other, as shown in Fig. 5.26(*b*). The number of such dipoles is equal to the number of turns in the solenoid. The north pole of one touches the south of the adjacent one. The opposite poles neutralise each other except at the ends. Thus, a current carrying solenoid can be replaced by just a single south pole and a single north pole, separated by a distance equal to the length of the solenoid. Hence a current carrying solenoid is equal to a bar magnet as shown in Fig. 5.26(*c*).

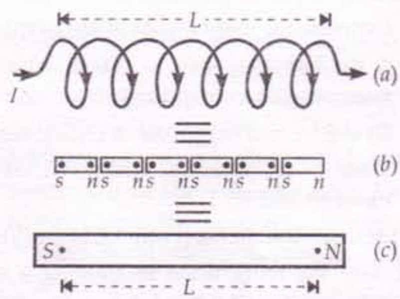


Fig. 5.26 A solenoid as an equivalent bar magnet.

A bar magnet and a finite solenoid produce similar magnetic field patterns, as shown in Fig. 5.24 and Fig. 5.25 respectively. It may be noted that the magnetic field inside the solenoid is in direction opposite to that we expect on the basis of the above pole model ($N \rightarrow S$).

18. *Explain how is a bar magnet equivalent to a current carrying solenoid.*

A bar magnet as an equivalent solenoid. We can explain this by ***Ampere's hypothesis*** according to which *all magnetic effects are produced by current-loops.* The electrons in an atom keep on revolving around its nucleus and hence set up electric currents. These atomic currents are equivalent to small circular current-loops. In a magnet, these current-loops are arranged parallel to each other and have currents in the same sense.

Figure 5.27 shows the atomic current loops in a cross-section of a cylindrical bar magnet. At any point inside the magnet, the currents from the adjacent loops cancel each other and hence the net current is zero. But there is a net current on the surface. Due to this surface current, the bar magnet is equivalent to a closely-wound, current carrying solenoid. Hence a bar magnet produces a magnetic field similar to the solenoid.

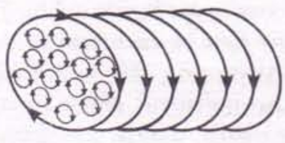


Fig. 5.27 A bar magnet as an equivalent solenoid.

It may be noted here that at the ends of the magnet, the current loops behave differently from those inside the magnet. As a result, the magnetic poles are located slightly inside the bar magnet. That is why the magnetic length of a bar magnet is slightly less than its geometrical length.

5.15 GAUSS'S LAW IN MAGNETISM

19. *State Gauss's law in magnetism. What are its important consequences ?*

Gauss's law in magnetism. Gauss's law in electrostatics states that the surface integral of the electrostatic field $\vec{E}$ over a closed surface S is equal to $1/\varepsilon_0$ times the total charge q enclosed by the surface S, *i.e.*,

$$\oint_S \vec{E}\cdot\vec{dS} = \frac{q}{\varepsilon_0}$$

Suppose that the closed surface S encloses an electric dipole which consists of two equal and opposite charges. Then the total charge enclosed by S is zero so that the surface integral of the electrostatic field of a dipole over the closed surface is also zero, *i.e.*,

$$\oint_S \vec{E}_{\text{dipole}}\cdot\vec{dS} = 0$$

Now a magnetic field is produced only by a magnetic dipole because isolated magnetic poles do not exist, so the above equation for a magnetic field can be written as

$$\oint_S \vec{B}\cdot\vec{dS} = 0$$

This is ***Gauss's law in magnetism*** *which states that the surface integral of a magnetic field over a closed surface is always zero.* But the surface integral of a magnetic field over a surface gives magnetic flux through that surface. So Gauss's law in magnetism can also be stated as follows :

The net magnetic flux through a closed surface is zero.

Consequences of Gauss's law :

1. Gauss's law indicates that *there are no sources or sinks of magnetic field inside a closed surface.* So there is no point at which the field lines start or there is no point at which the field lines terminate. In other words, there are no free magnetic charges. Hence *isolated magnetic poles (also called monopoles) do not exist.*
2. The magnetic poles always exist as unlike pairs of equal strengths.
3. If a number of magnetic lines of force enter a closed surface, then an equal number of lines of force must leave that surface.

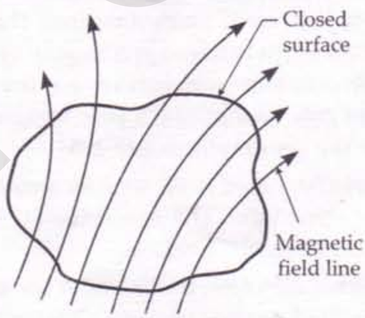


Fig. 5.28 Magnetic field lines never terminate.

For Your Knowledge

- Gauss's law of magnetism formally expresses the fact that *magnetic monopoles do not exist.* Hence the most elementary magnetic element is a magnetic dipole or a current loop. All magnetic phenomena can be explained in terms of an arrangement of magnetic dipoles and/or current loops.
- **Basic difference between electric and magnetic lines of force.** An important consequence of the fact that magnetic monopoles do not exist is that magnetic lines of force are continuous and form closed loops. They do not start or end at a point. In contrast, the electric lines of force start from a positive charge and end on a negative charge or they fade out at infinity in case of isolated charges.

5.16 MAGNETIC FIELD OF THE EARTH

20. *Give some experimental evidences which support the existence of earth's magnetic field.*

Magnetic field of the earth. Earth is a powerful natural magnet. Its magnetic field is present everywhere near the earth's surface. This field can be approximated to the field of a magnetic dipole of dipole moment 8.0×10^{22} Am^2 assumed to be located at the centre of the earth. The axis of the dipole makes an angle of about 20° with the axis of rotation of the earth. The *magnetic north pole* N_m of the earth lies somewhere near the *geographic south pole* S_g while the *magnetic south pole* S_m lies somewhere near the *geographic north pole* N_g. The magnitude of the magnetic field on the earth's surface is typically about 10^{-4}T which is equal to 1 *gauss* (G). A gauss is also often called an *oersted*. Thus the earth's magnetic field is of the order of 1 oersted.

The branch of physics that deals with the study of earth's magnetism is called ***terrestrial magnetism*** or ***geomagnetism***.

Experimental evidences in support of earth's magnetism :

1. *A freely suspended magnetic needle comes to rest roughly in north-south direction*. This suggests that the earth behaves as a huge magnet with its south pole lying somewhere near the geographic north pole and its north pole lying somewhere near the geographic south pole.
2. *An iron bar buried in the earth becomes weak magnet* after some time. The magnetism is induced by earth's magnetic field.
3. *Existence of neutral points near a bar magnet* indicates the presence of earth's magnetic field. At these points, the magnetic field of the magnet is cancelled by the earth's magnetic field.

5.17 ORIGIN OF EARTH'S MAGNETIC FIELD

21. *Give a brief account of different theories regarding the source of earth's magnetism.*

Origin of earth's magnetic field. The magnetic field of the earth is approximately like that of a *giant bar magnet* embedded deep inside the earth. Many theories have been proposed about the cause of earth's magnetism from time to time. Some of these are mentioned below :

1. In 1600, *William Gilbert* in his book '*De Magnete*' first suggested that the earth behaves as a bar magnet and its magnetism is due to the presence of magnetic material at its centre, which could be a permanent magnet. However, the core of the earth is so hot that a permanent magnet cannot exist there.

2. *Prof. Blackett* suggested that the earth's magnetism is due to the rotation of the earth about its own axis. Every substance is made of charged particles such as protons and electrons. As these particles rotate along with the earth, they cause circulating currents which, in turn, magnetise the earth.

3. Cosmic rays cause the ionisation of gases in the earth's atmosphere. As the earth rotates, strong electric currents are set up due to the movement of the charged ions. These currents may be the source of earth's magnetism.

4. According to *Sir E. Bullard* (U.K.) and *W.M. Elaster* (U.S.A.), there are large deposits of ferromagnetic materials like iron, nickel, etc. in the core of the earth. The core of the earth is very hot and molten. *The circulating ions in the highly conducting liquid region of the earth's core form current loops and hence produce a magnetic field*. At present, this hypothesis seems most probable because our moon, which has no molten core, has no magnetic field. Venus, which has a slower rate of rotation, has a weaker magnetic field while Jupiter, with a faster rate of rotation has a stronger magnetic field.

The changes in the earth's magnetic field are so complicated and irregular that the exact cause of earth's magnetism is yet to be known.

5.18 SOME DEFINITIONS IN CONNECTION WITH EARTH'S MAGNETISM

22. *Define the terms geographic axis, magnetic axis, magnetic equator, magnetic meridian and geographic meridian in connection with geomagnetism.*

Some definitions in connection with earth's magnetism. Fig. 5.29 shows the magnetic lines of force around the earth.

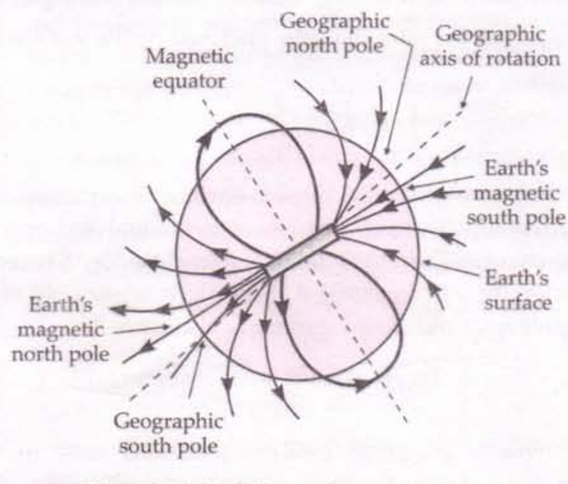


Fig. 5.29 Magnetic field of the earth.

1. Geographic axis. *The straight line passing through the geographical north and south poles of the earth is called its geographic axis. It is the axis of rotation of the earth.*

2. Magnetic axis. *The straight line passing through the magnetic north and south poles of the earth is called its magnetic axis.*

The magnetic axis of the earth makes an angle of nearly 20° with the geographic axis. At present, the magnetic south pole S_m is located at a point in Northern Canada at a latitude of 70.5°N and a longitude of 96°W. The magnetic north pole N_m is located diametrically opposite to S_m *i.e.*, at a latitude of 70.5°S and a longitude of 84°E. The magnetic poles are nearly 2000 km away from the geographic poles. The magnetic equator intersects the geographic equator at longitudes of 6°W and 174°E.

3. Magnetic equator. *It is the great circle on the earth perpendicular to the magnetic axis.*

4. Magnetic meridian. *The vertical plane passing through the magnetic axis of a freely suspended small magnet is called magnetic meridian. The earth's magnetic field acts in the direction of the magnetic meridian.*

5. Geographic meridian. *The vertical plane passing through the geographic north and south poles is called geographic meridian.*

5.19 ELEMENTS OF EARTH'S MAGNETIC FIELD

23. *What are the elements of earth's magnetic field ? Explain their meanings. Show these elements in a labelled diagram and deduce various relations between them.*

Elements of earth's magnetic field. *The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field.* They are declination, dip and horizontal component of earth's magnetic field.

1. Magnetic declination. *The angle between the geographical meridian and the magnetic meridian at a place is called the magnetic declination* (α) *at that place.*

Magnetic declination arises because the magnetic axis of the earth does not coincide with its geographic axis.

To determine magnetic declination at a place, set up a compass needle that is free to rotate in a horizontal plane about a vertical axis, as shown in Fig. 5.30. The angle α that this needle makes with the geographic north-south ($N_g - S_g$) direction is the magnetic declination. By knowing declination, we can determine the vertical plane in which the earth's magnetic field lies. In India, the value of α is small. It is 0°41′ E for Delhi and 0°58′ W for Mumbai. This means that the *N*-pole of a compass needle almost points in the direction of geographic north.

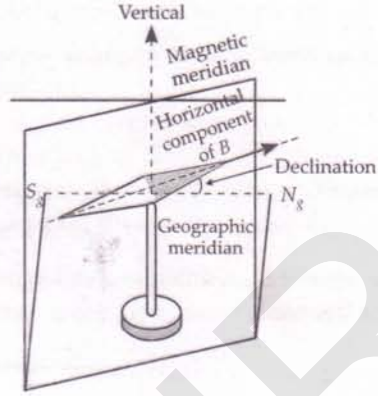


Fig. 5.30 Determination of declination at a place.

2. Angle of dip or magnetic inclination. *The angle made by the earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called angle of dip* (δ) *at any place.*

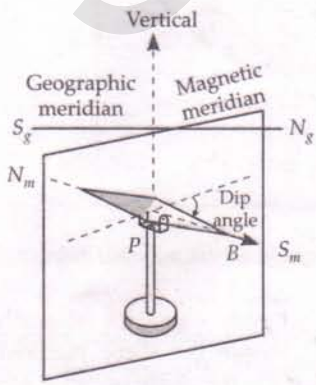


Fig. 5.31 Determination of dip at a place.

The angle of dip is different at different places on the surface of the earth. Consider a *dip needle,* which is just another compass needle but pivoted horizontally so that it is free to rotate in a vertical plane coinciding with the magnetic meridian. It orients itself so that its N-pole finally points exactly in the direction of the earth's total magnetic field $\vec{B}$. The angle between the horizontal and the final direction of the dip needle gives the angle of dip at the given location.

At the magnetic equator, the dip needle rests horizontally so that the *angle of dip is zero at the magnetic equator.* The dip needle rests vertically at the magnetic poles so that *the angle of dip is 90° at the magnetic poles.* At all other places, the dip angle lies between 0° and 90°.

3. Horizontal component of earth's magnetic field. *It is the component of the earth's total magnetic field $\vec{B}$ in the horizontal direction in the magnetic* *meridian.* If δ is the angle of dip at any place, then the horizontal component of earth's field $\vec{B}$ at that place is given by

$$B_H = B \cos \delta$$

At the magnetic equator, $\delta = 0°$, $B_H = B \cos 0° = B$

At the magnetic poles, $\delta = 90°$, $B_H = B \cos 90° = 0$

Thus the value of B_H is different at different places on the surface of the earth.

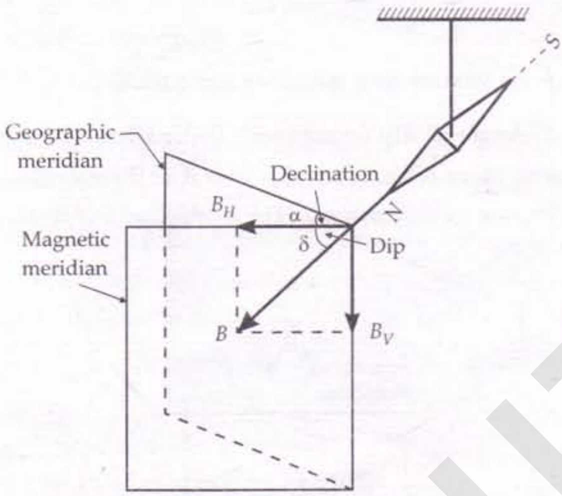


Fig. 5.32 Elements of earth's magnetic field.

Relations between elements of earth's magnetic field. Fig. 5.32 shows the three elements of earth's magnetic field. If δ is the angle of dip at any place, then the horizontal and vertical components of earth's magnetic field $\vec{B}$ at that place will be

$$B_H = B \cos \delta \qquad ...(1)$$

and $$B_V = B \sin \delta$$

$$\therefore \quad \frac{B_V}{B_H} = \frac{B \sin \delta}{B \cos \delta}$$

or $$\frac{B_V}{B_H} = \tan \delta \qquad ...(2)$$

Also

$$B_H^2 + B_V^2 = B^2(\cos^2 \delta + \sin^2 \delta) = B^2$$

or $$B = \sqrt{B_H^2 + B_V^2} \qquad ...(3)$$

Equations (1), (2) and (3) are the different relations between the elements of earth's magnetic field. By knowing the three elements, we can determine the magnitude and direction of the earth's magnetic field at any place.

For Your Knowledge

➢ **Magnetic maps.** *These are the detailed charts which indicate on the world map the lines passing through all such places where one of the three magnetic elements has the same value.* Three types of lines are drawn on such maps. These are

1. **Isogonic lines.** *The lines joining the places of equal declination are called isogonic lines.* The line of zero declination is called *agonic line.*
2. **Isoclinical lines.** *The lines joining the places of equal dip or inclination are called isoclinical lines.* The line of zero dip is called *aclinic line or magnetic equator.* The points of 90° dip are called magnetic poles. The magnetic equator crosses the geographic equator twice once in Atlantic and then in Pacific ocean.
3. **Isodynamic lines.** *The lines joining the places having the same value of the horizontal component of earth's magnetic field are called isodynamic lines.* The horizontal component is zero at poles and maximum at the magnetic equator.

5.20 GLOBAL VARIATIONS IN THE EARTH'S MAGNETIC FIELD*

24. *Describe the variations of earth's magnetic field from place to place.*

Global variations in the earth's magnetic field. Earth's magnetic field changes both in magnitude and direction from place to place. Some of the noticeable global variations are as follows :

1. The magnitude of the magnetic field on earth's surface is small, nearly 4×10^{-5} T.
2. Still smaller is the background field of our own galaxy, the Milky Way, being about 2 pT *i.e.,* 2×10^{-12} T.
3. If we assume that the earth's field is due to dipole of 8.0×10^{22} Am^2 located at its centre, then the earth's magnetic field will be less than 1 μT (10^{-6}T) at a distance of 5 times the radius of the earth *i.e.,* at about 32,000 km. Upto this distance, the magnetic field is entirely governed by the earth.
4. At distances greater than 32,000 km, the pattern of the earth's magnetic field gets severely distorted by the *solar wind.*
5. Solar wind causes ionisation of atmosphere near the magnetic poles of the earth. This in turn causes beautiful displays of colours high up in the sky and is known as *aurora.*

25. *What is solar wind ? How does it affect earth's magnetic field ?*

Solar wind. *The solar wind is a stream of hot charged ions, composed of equal numbers of protons and electrons continuously flowing radially outward from the sun with a speed of approximately 400 km/s.* A long magneto tail stretches out for several thousand earth diameters in a direction away from the sun.

At distances greater than 32,000 km, the dipole field pattern of the earth's magnetic field gets severely distorted by the solar wind, as shown in Fig. 5.33.

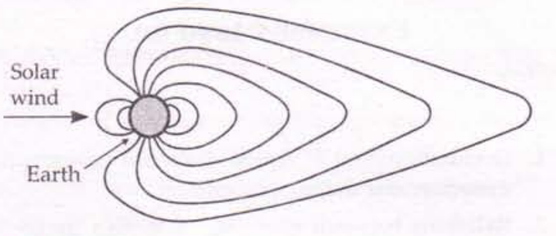


Fig. 5.33 Distortion of earth's magnetic field by the solar wind.

26. *What are aurora borealis and aurora australis ? Can this effect be seen anywhere in India ?*

Aurora borealis and aurora australis. This is a spectacular display of light seen in the night sky at high altitudes, occurring most frequently near the earth's magnetic poles. The displays of aurora appear as gaint curtains high up in the atmosphere. The aurora is caused when the charged particles of the solar wind get attracted by the magnetic poles of the earth and there they ionise the atmospheric atoms or molecules. The aurora in the northern hemisphere is called ***aurora borealis*** or ***northern lights*** and the aurora in southern hemisphere is called ***aurora australis*** or ***southern lights***.

5.21 TEMPORAL VARIATIONS IN THE EARTH'S MAGNETIC FIELD*

27. *Describe the variations of earth's magnetic field that have occurred with the passage of time.*

Temporal variations in the earth's magnetic field. The earth's magnetic field changes both in magnitude and direction as time passes. These changes are of two types :

(*i*) ***Short term changes.*** The position of the magnetic poles of the earth keeps shifting slowly with the passage of time. In a period of 240 years, from 1580 to 1820, the magnetic declination at London has changed by 35°. The magnetic south pole in the northern Arctic region of Canada has been found to shift in the north-west direction at the rate of 10 km per year in recent times.

(*ii*) ***Long term changes.*** The changes in earth's magnetic field over long term or geological time scales are interesting. The studies of basalt reveal that earth's magnetic field reverses its direction every million years or so. This means that once in a million years or so, the currents in earth's core cool down, come to a halt and then pick up speed in the opposite direction.

Basalt which contains iron, is emitted during volcanic activity on the ocean floor. As it cools, it solidifies and provides a picture of earth's magnetic field. Its age can be determined by other means.

5.22 NEUTRAL POINT

28. *Define neutral point. How will you find the magnetic moment of a bar magnet by locating its neutral points, when the magnet is placed with its north pole towards (i) north pole of the earth and (ii) south pole of the earth ?*

Neutral point. *It is the point where the magnetic field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field. The resultant magnetic field at the neutral point is zero.* If a compass needle is placed at such a point, it can stay in any position.

(*i*) ***Magnet placed in the magnetic meridian with its north pole pointing north.*** Fig. 5.34 shows the magnetic lines of force of a bar magnet placed in the magnetic meridian with its north-pole pointing towards the geographic north of the earth. The fields due to the magnet and the earth are in same directions at points on the axial line and are in opposite directions at points on the equatorial line. So the resultant field is stronger at axial points and weaker at equatorial points. The two neutral points P and Q lie on the equatorial line.

Let

r = distance of each neutral point from the centre of the magnet

$2l$ = length of the magnet

m = dipole moment of the magnet

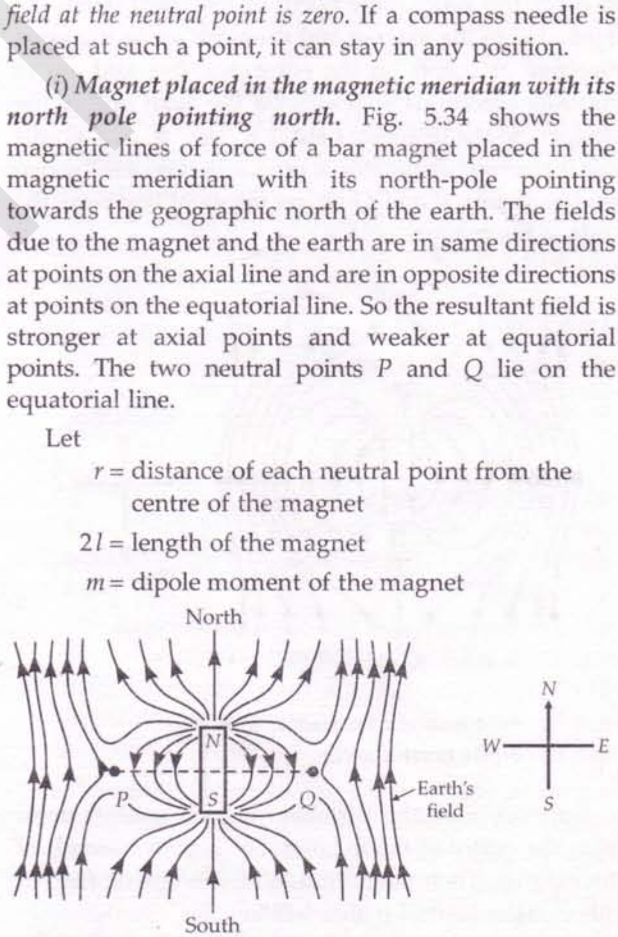


Fig. 5.34 Field lines of a bar magnet with its N-pole towards north.

Then megnetic field strength at each neutral point is

$$B_{\text{equa}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}}$$

For a short magnet, $l << r$, therefore,

$$B_{\text{equa}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

At the neutral point, the field of the magnet is balanced by the horizontal component B_H of the earth's magnetic field so that

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

Knowing r and B_H, the value of the magnetic dipole moment m can be determined.

(*ii*) ***Magnet placed in the magnetic meridian with its south-pole pointing north.*** Fig. 5.35 shows the magnetic lines of force of a bar magnet placed in the magnetic meridian with its south-pole pointing towards the geographic north of the earth. Here the fields due to the magnet and the earth are in the same direction at points on the equatorial line and are in opposite directions at points on the axial line of the magnet. So the resultant field is weaker at axial points and is stronger at equatorial points. In this case the two neutral points P and Q lie on the axial line near the ends of the magnet.

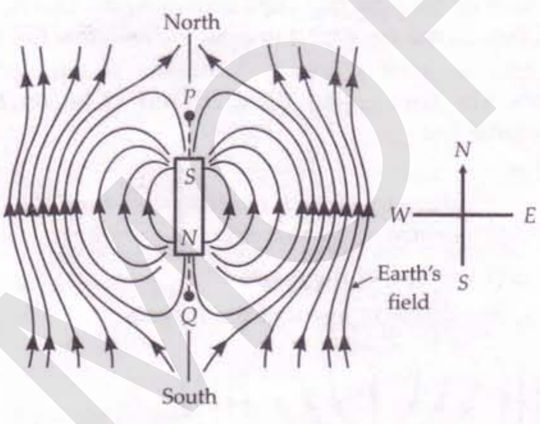


Fig. 5.35 Field lines of a bar magnet with S-pole towards north.

Suppose r be the distance of each neutral point from the centre of the magnet. Let $2l$ be the length of the magnet. Then magnitude of the magnetic field at either of the neutral points will be

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2}$$

For a short magnet, $l << r$, therefore

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

Again, at the neutral point, the field of the magnet is balanced by the horizontal component B_H of the earth's magnetic field, so we have

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

Knowing the values of r and B_H, the magnetic dipole moment m of the magnet can be determined.

Examples based on

Earth's Magnetism and Neutral Points

Formulae Used

1. Declination (α) = Angle between geographic meridian and magnetic meridian.
2. Relations between elements of earth's magnetic field are

$$B_H = B\cos\delta \quad \text{and} \quad B_V = B\sin\delta$$

$$\frac{B_V}{B_H} = \tan\delta \quad \text{and} \quad B = \sqrt{B_H^2 + B_V^2}$$

3. For a magnet placed with its N-pole pointing north, neutral points lie at its equatorial line.

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}}$$

$$\approx \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} \quad \text{[for a short magnet]}$$

4. For a magnet placed with its N-pole pointing south, neutral points lie on its axial line.

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} \quad \text{[for a short magnet]}$$

Units Used

Magnetic fields B, B_H and B_V are in tesla, distances r and l in metre, magnetic moment m in JT^{-1} or Am^2, angle α and δ are in degrees.

Constant Used

$$\mu_0 = 4\pi \times 10^{-7} \text{TmA}^{-1}.$$

Example 24. *The declination at a place is 15° west of north. In which direction should a ship be steered so that it reaches a place due east ?*

Solution. As the ship is to reach a place due east *i.e.*, along OP (Fig. 5.36), so it should be steered at angle of $15° + 90° = 105°$ with the direction of the compass needle.

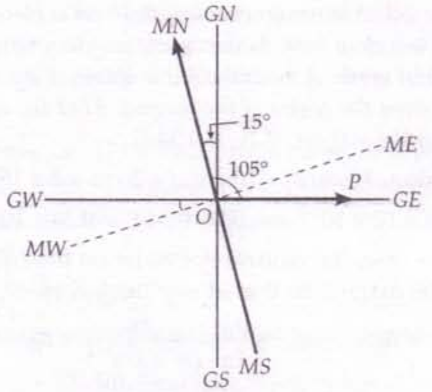


Fig. 5.36

Example 25. *A ship is to reach a place 10° south of west. In which direction should it be steered if the declination at the place is 18° west of north ?*

Solution. As the ship is to reach a place 10° south of west *i.e.*, along *OP* (Fig. 5.37), so it should be steered west of magnetic north at angle of 90 − 18 + 10 = 82°.

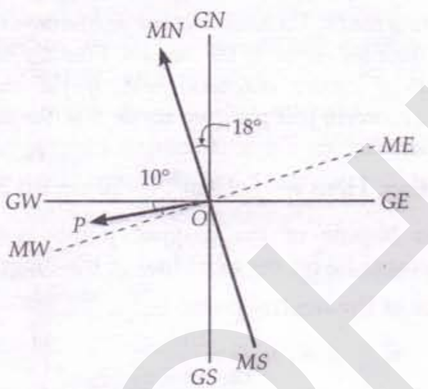


Fig. 5.37

Example 26. *In the magnetic meridian of a certain place, the horizontal component of the earth's magnetic field is 0.26 G and the dip angle is 60°. What is the magnetic field of the earth in this location ?* [NCERT]

Solution. Here $B_H = 0.26$ G, $\delta = 60°$

As $B_H = B\cos\delta$

$$\therefore \quad B = \frac{B_H}{\cos\delta} = \frac{0.26\text{ G}}{\cos 60°} = \frac{0.26\text{ G}}{0.5} = \mathbf{0.52\ G.}$$

Example 27. *A compass needle whose magnetic moment is 60 Am² pointing geographical north at a certain place where the horizontal component of earth's magnetic field is 40 μWb/m² experiences a torque of 1.2×10^{-3} Nm. What is the declination of the place ?* [Roorkee 82]

Solution. In stable equilibrium, a compass needle points along magnetic north and experiences no torque. When it is turned through declination α, it points along geographic north and experiences torque,

$$\tau = mB\sin\alpha$$

$$\therefore \quad \sin\alpha = \frac{\tau}{mB} = \frac{1.2\times10^{-3}}{60\times40\times10^{-6}} = \frac{1}{2}$$

or $\alpha = \mathbf{30°}$.

Example 28. *The horizontal and vertical components of earth's field at a place are 0.22 gauss and 0.38 gauss respectively. Calculate the angle of dip and resultant intensity of earth's field.* [Haryana 93 ; Himachal 94]

Solution. Here $B_H = 0.22$ G, $B_V = 0.38$ G

Now $\tan\delta = \frac{B_V}{B_H} = \frac{0.38}{0.22} = 1.7272$

∴ Angle of dip, $\delta = 59°56'$

Resultant magnetic field of the earth is

$$B = \sqrt{B_H^2 + B_V^2} = \sqrt{0.22^2 + 0.38^2} = \mathbf{0.427\ G.}$$

Example 29. *If the horizontal component of earth's magnetic field at a place where the angle of dip is 60° is 0.4×10^{-4}T, calculate the vertical component and the resultant magnetic field at that place.* [CBSE OD 97C ; Haryana 01]

Solution. Here $\delta = 60°$, $B_H = 0.4\times10^{-4}$ T

$$B_V = B_H\tan\delta = 0.4\times10^{-4}\tan 60°$$
$$= 0.4\times10^{-4}\times\sqrt{3} = \mathbf{0.69\times10^{-4}\ T.}$$

Resultant magnetic field,

$$B = \frac{B_H}{\cos\delta} = \frac{0.4\times10^{-4}}{\cos 60°} = \mathbf{0.8\times10^{-4}\ T.}$$

Example 30. *If δ_1 and δ_2 be the angles of dip observed in two vertical planes at right angles to each other and δ is the true angle of dip, prove that $\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$* [Punjab 96]

Solution. Let B_H and B_V be the horizontal and vertical components of earth's magnetic field $\vec{B}$. Since δ is the true angle of dip, therefore

$$\tan\delta = \frac{B_V}{B_H}$$

or $$\cot\delta = \frac{B_H}{B_V} \quad ...(1)$$

As shown in Fig. 5.38, suppose planes 1 and 2 are two mutually perpendicular planes and respectively make angles θ and 90° − θ with the magnetic meridian. The vertical components of earth's magnetic field remain same in the two planes but the effective horizontal components in the two planes will be

$$B_1 = B_H\cos\theta \quad \text{and} \quad B_2 = B_H\sin\theta$$

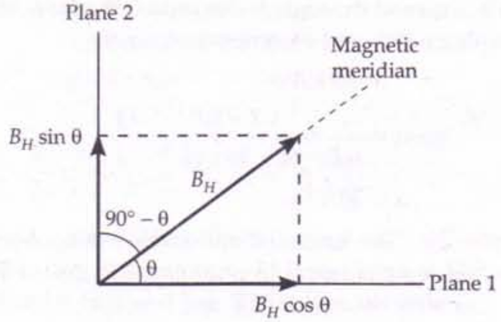


Fig. 5.38

The angles of dip δ_1 and δ_2 in the two planes are given by

$$\tan\delta_1 = \frac{B_V}{B_1} = \frac{B_V}{B_H \cos\theta}$$

or $$\cot\delta_1 = \frac{B_H \cos\theta}{B_V} \quad ...(2)$$

$$\tan\delta_2 = \frac{B_V}{B_2} = \frac{B_V}{B_H \sin\theta}$$

or $$\cot\delta_2 = \frac{B_H \sin\theta}{B_V} \quad ...(3)$$

From equations (2) and (3), we have

$$\cot^2\delta_1 + \cot^2\delta_2 = \frac{B_H^2}{B_V^2}(\cos^2\theta + \sin^2\theta) = \frac{B_H^2}{B_V^2}$$

or $$\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta. \quad \text{[Using equation (1)]}$$

Example 31. *A dip circle shows an apparent dip of 60° at a place where the true dip is 45°. If the dip circle is rotated through 90°, what apparent dip will it show ?*

Solution. Here $\delta_1 = 60°$ and $\delta = 45°$.

As $\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$

$\therefore$ $\cot^2 45° = \cot^2 60° + \cot^2\delta_2$

or $$\cot^2\delta_2 = (1)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{2}{3}$$

or $\cot\delta_2 = 0.816$

$\therefore$ $\delta_2 = \mathbf{51°}$.

Example 32. *True value of dip at a place is 45°. The plane of the dip circle is turned through 60° from the magnetic meridian. Find the apparent value of dip.*

Solution. Here $\delta = 45°$, $\theta = 60°$, $\delta' = ?$

$$\tan\delta' = \frac{B_V}{B'_H} = \frac{B_H \tan\delta}{B_H \cos\theta}$$

$$= \frac{\tan\delta}{\cos\theta} = \frac{\tan 45°}{\cos 60°} = 2$$

$\therefore$ Apparent dip, $\delta' = \mathbf{63.4°}$.

Example 33. *A bar magnet of length 10 cm is placed in the magnetic meridian with its north pole pointing towards the geographical north. A neutral point is obtained at a distance of 12 cm from the centre of the magnet. Find the magnetic moment of the magnet, if $B_H = 0.34$ G.*

Solution. Here, $2l = 10$ cm, $l = 5$ cm $= 5 \times 10^{-2}$ m

$r = 12$ cm $= 12 \times 10^{-2}$ m, $B_H = 0.34$ G $= 0.34 \times 10^{-4}$ T

In this case, the neutral points lie on the equatorial line of the magnet so that at any neutral point,

$$B_{equa} = B_H \quad \text{or} \quad \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}} = B_H$$

$\therefore$ Magnetic moment

$$m = B_H \cdot \frac{4\pi}{\mu_0} \cdot (r^2 + l^2)^{3/2}$$

$$= 0.34 \times 10^{-4} \times \frac{1}{10^{-7}} [5^2 + 12^2]^{3/2} (10^{-4})^{3/2}$$

$$= \mathbf{0.747\ JT^{-1}}.$$

Example 34. *The magnetic moment of a short bar magnet is 1.6 Am^2. It is placed in the magnetic meridian with north pole pointing south. The neutral point is obtained at distance of 20 cm from the centre of the magnet. Find the horizontal component of earth's magnetic field. If the magnet be reversed, i.e., north pole pointing north, find the position of neutral point.*

Solution. Here $m = 1.6$ Am^2, $r = 20$ cm $= 0.20$ m

When N-pole of the magnet points south, the neutral points lie on the axial line of the magnet.

Hence at the neutral point,

$$B_{axial} = B_H = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

or $$B_H = \frac{10^{-7} \times 2 \times 1.6}{(0.20)^3} = \mathbf{4 \times 10^{-5}\ T}$$

When the magnet is reversed, its north pole points north. The neutral points will lie on the equatorial line of the magnet. Hence

$$B_{equa} = B_H = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

or $$r^3 = \frac{\mu_0}{4\pi} \cdot \frac{m}{B_H} = \frac{10^{-7} \times 1.6}{4 \times 10^{-5}} = 4.0 \times 10^{-3}\ \text{m}^3$$

$\therefore$ $r = (4.0 \times 10^{-3})^{1/3}$

$= 1.6 \times 10^{-1}$ m $=$ **16 cm.**

Example 35. *A magnet placed in the magnetic meridian with its north pole pointing north of the earth produces a neutral point at a distance of 0.15 m from either pole. It is then broken into two equal pieces and one such piece is placed in a similar position. Find the position of the neutral point.*

Solution. Here the neutral points lie on the equatorial line of the magnet at distance x from each of the two poles.

$$\therefore \quad B_H = B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{x^3}$$

When the magnet is broken into two parts, its pole strength remains unchanged.

Original magnetic moment,

$$m = q_m \times 2l$$

Magnetic moment of each part,

$$m' = q_m \times \frac{2l}{2} = \frac{m}{2}$$

$$\therefore \quad B_H = \frac{\mu_0}{4\pi} \cdot \frac{m/2}{x'^3}$$

Hence $$\frac{\mu_0}{4\pi} \cdot \frac{m}{x^3} = \frac{\mu_0}{4\pi} \cdot \frac{m}{2x'^3}$$

or $$2x'^3 = x^3$$

or $$x' = \frac{x}{2^{1/3}}$$

or $x' = \dfrac{0.15}{1.26} = $ **0.119 m,** from each pole.

Example 36. *The magnetic field at a point on the magnetic equator is* 3.1×10^{-5}*T. Taking the radius of the earth equal to 6400 km, find the magnetic moment of the assumed dipole at the earth's centre.*

Solution. Any point on the magnetic equator lies in the broad side on position of the assumed magnetic dipole. Hence

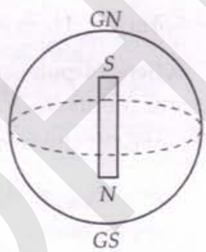


Fig. 5.39

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{R^3}$$

or $$m = \frac{4\pi}{\mu_0} \cdot B_{equa} \; R^3$$

$$= 10^7 \times 3.1 \times 10^{-5} \times (6400 \times 10^3)^3$$

$$= \mathbf{8.1 \times 10^{22} \; Am^2}.$$

Example 37. *The earth's magnetic field at the equator is approximately 0.4 G. Estimate the earth's dipole moment. Radius of the earth = 6400 km* [NCERT]

Solution. Here $B_H = B_{equa} = \dfrac{\mu_0 m}{4\pi r^3}$

or $$0.4 \times 10^{-4} = \frac{10^{-7} \times m}{(6.4 \times 10^6)^3}$$

or $$m = \mathbf{1.04 \times 10^{23} \; Am^2}.$$

Example 38. *A short bar magnet is placed in a horizontal plane with its axis in the magnetic meridian. Null points are found on its equatorial line (i.e., its normal bisector) at 12.5 cm from the centre of the magnet. The earth's magnetic field at the place is 0.38 G and the angle of dip is zero.*

(*i*) *What is the total magnetic field at points on the axis of the magnet located at the same distance (12.5 cm) as the null-points from the centre ?*

(*ii*) *Locate the null points when the magnet is turned around by 180°.*

Assume that the length of the magnet is negligible as compared to the distance of the null-points from the centre of the magnet.

Solution. (*a*) At the neutral point on the equatorial line of a short magnet, we have

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = B_H$$

Magnetic field of the magnet on its axial line at the same distance will be

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = 2\,B_H = 2 \times 0.38 = 0.76 \text{ G}$$

At any point on the axial line, B_H and B_{axial} are in the same direction. So total magnetic field,

$$B = B_{axial} + B_H = 0.76 + 0.38 = \mathbf{1.14 \; G}.$$

(*b*) When the magnet is turned through 180°, the neutral points lie on the axial line.

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{x^3} = B_H$$

But $$B_H = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} \qquad \therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{x^3}$$

or $$x^3 = 2r^3$$

or $$x = (2)^{1/3}\, r = 1.26 \times 12.5 \text{ cm} = \mathbf{15.75 \; cm}.$$

Problems For Practice

1. A ship is sailing due west according to Mariner's compass. If the declination of the place is 15° east of north, what is true direction of the ship ?
   (Ans. 75° west of north)

2. A ship is sailing due east according to Mariner's compass. If the declination of the place is 18° east of north, what is the true direction of the ship ?
   (Ans. 18° south of east)

3. The horizontal component of earth's magnetic field is 0.2 G and total magnetic field is 0.4 G. Find angle of dip. **[Haryana 96]** (Ans. 60°)

4. Calculate earth's magnetic field at a place, where the angle of dip is 60° and vertical component of earth's field is 0.40 G. (Ans. 0.462 G)

5. A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip down 60° with the horizontal. The horizontal

component of the earth's magnetic field at the place known to be 0.4 G. Determine the magnitude of the earth's magnetic field at the place. **[CBSE F 11]**

(Ans. 0.8 G)

6. The vertical and horizontal components of earth's magnetic field at a place are 0.2 G and 0.3464 G respectively. Calculate the angle of dip and earth's magnetic field at that place. (Ans. 30°, 0.4 G)

7. A vertical wire in which current is flowing produces a neutral point with the earth's horizontal field at a distance of 5 cm from the wire in air. What is current, if $B_H = 0.18 \times 10^{-4}$ T ? (Ans. 4.5 A)

8. A compass needle whose magnetic moment is 60 Am^2 pointing geographical north at a certain place, where the horizontal component of earth's magnetic field is 40 μT, experiences a torque of 1.2×10^{-3} Nm. What is the declination at that place ? (Ans. 30°)

9. A magnetic needle free to rotate about the vertical direction (compass) points 3.5° west of the geographic north. Another magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 18° with the horizontal. The magnitude of the horizontal component of the earth's magnetic field at the place is known to be 0.40 G. What is the direction and magnitude of the earth's magnetic field at the place ?

(Ans. 0.42 G, directed at an angle of 18° with horizontal in the magnetic meridian towards the ground)

10. The true dip at a place is 30°. What is the apparent dip when the dip circle is turned 60° out of the magnetic meridian ? (Ans. 49°7′)

11. The values of the apparent angles of dip in two planes at right angles to each other are 30° and 45°. Calculate the true value of the angle of dip at the place. (Ans. 26.6°)

12. A dip circle lying initially in the magnetic meridian is rotated through an angle θ in the horizontal plane. Show that the tangent of the angle of dip is increased in the ratio sec θ : 1.

13. A short bar magnet of magnetic moment 0.5 J T^{-1} is placed with its magnetic axis in the magnetic meridian, with its north pole pointing geographical north. A neutral point is obtained at a distance of 0.1 m from the centre of the magnet. Find the horizontal component of the earth's magnetic field. (Ans. 10^{-4} T)

14. A bar magnet 30 cm long is placed in the magnetic meridian with its north pole pointing geographical south. The neutral point is found at a distance of 30 cm from its centre. Calculate the pole strength of the magnet. Given $B_H = 0.34$ G. (Ans. 8.61 Am)

15. A neutral point is found on the axis of a bar magnet at a distance of 10 cm from its one end. If the length of the magnet be 10 cm and $B_H = 0.3$ G, find the magnetic moment of the magnet. (Ans. 0.012 Am^2)

16. A magnet placed in the north pointing north position, balances the earth's magnetic field at a point, which is 27 cm from either pole. If it is broken into three pieces and one such piece is similarly placed, find the position of the neutral point.

(Ans. 18.73×10^{-2} m, from either pole)

HINTS

4. $\therefore \quad B = \dfrac{B_V}{\sin \delta} = \dfrac{0.40}{\sin 60°} = \dfrac{0.40}{0.866} = \mathbf{0.462\ G.}$

5. Here, $\delta = 60°,\ B_H = 0.4$ G, $B = ?$

$$B = \frac{B_H}{\cos \delta} = \frac{0.4}{\cos 60°} = \frac{0.4}{0.5} = \mathbf{0.8\ G.}$$

7. As $\dfrac{\mu_0 I}{2\pi r} = B_H$

$$\therefore \quad I = \frac{2\pi r\, B_H}{\mu_0} = \frac{2\pi \times 5 \times 10^{-2} \times 0.18 \times 10^{-4}}{4\pi \times 10^{-7}} = \mathbf{4.5\ A.}$$

8. $\tau = m\, B_H \sin \alpha$

$\therefore \quad 1.2 \times 10^{-3} = 60 \times 40 \times 10^{-6} \times \sin \alpha$

or $\quad \sin \alpha = \dfrac{1}{2} \quad \therefore \quad \alpha = \mathbf{30°.}$

9. Here 3.5° is the magnetic declination and 18° is the angle of dip.

As $\quad B_H = B \cos \delta$

$\therefore \quad B = B_H \sec \delta = 0.40 \sec 18°$

$= 0.40 \times 1.0514 = \mathbf{0.42\ G.}$

11. $\cot^2 \delta = \cot^2 \delta_1 + \cot^2 \delta_2$

$= \cot^2 30° + \cot^2 45° = 3 + 1 = 4$

$\cot \delta = 2 \quad \therefore \quad \delta = \mathbf{26.6°.}$

12. The true angle of dip δ is given by $\tan \delta = \dfrac{B_V}{B_H}$

When the dip circle is rotated through angle θ, the apparent angle of dip δ′ is given by

$$\tan \delta' = \frac{B_V}{B_H \cos \delta} = \tan \delta . \sec \theta \quad \therefore \quad \frac{\tan \delta'}{\tan \delta} = \mathbf{\sec \theta : 1.}$$

13. Use $B_H = \dfrac{\mu_0}{4\pi} \cdot \dfrac{m}{r^3}$.

14. Here the neutral points lie on the axial line.

$\therefore \quad B_{axial} = B_H \quad$ or $\quad \dfrac{\mu_0}{4\pi} \cdot \dfrac{2\,mr}{(r^2 - l^2)^2} = B_H$

or $\quad m = \dfrac{4\pi}{\mu_0} \cdot \dfrac{B_H (r^2 - l^2)^2}{2r}$

$$= \frac{1}{10^{-7}} \cdot \frac{0.34 \times 10^{-4}\,(0.30^2 - 0.15^2)^2}{2 \times 0.30} = \mathbf{2.582\ Am^2}$$

Pole strength, $q_m = \dfrac{m}{2l} = \dfrac{2.582}{0.30} = \mathbf{8.61\ Am.}$

15. Use $B_H = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2m}{r^3}$. Here $r = 10 + 5 = 15$ cm.

16. Proceed as in Example 35 on page 5.26.

5.23 SOME IMPORTANT TERMS USED TO DESCRIBE MAGNETIC PROPERTIES OF MATERIALS

29. *Define the terms magnetising field, magnetic induction, intensity of magnetisation, magnetising field intensity, magnetic permeability, relative permeability and magnetic susceptibility. Write the various relations among these quantities.*

1. **Magnetising field.** When a magnetic material is placed in a magnetic field, a magnetism is induced in it. *The magnetic field that exists in vacuum and induces magnetism is called magnetising field.* For example, consider a toroidal solenoid carrying current I and placed in vacuum. If the solenoid has n turns per unit length, then the magnetic field set up in the solenoid is given by

$$B_0 = \mu_0 n I$$

This field is called the *magnetising field* caused by the so called free current in the solenoid.

2. **Magnetic induction.** As shown in Fig. 5.40, suppose the toroidal solenoid is wound round a ring of magnetic material. Under the influence of field $\vec{B}_0$, the magnetic moments of the atomic current loops of the magnetic material tend to align themselves with or against the magnetising field B_0. This gives rise to a net current on the surface of the material and is called magnetisation surface current I_M as shown in Fig. 5.41.

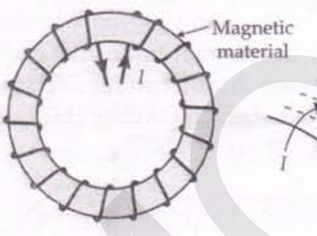


Fig. 5.40 Magnetic field in a magnetic material.

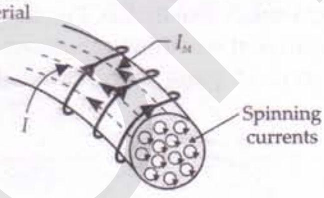


Fig. 5.41 Magnetising surface current.

This current induces magnetic field $\vec{B}_M$ inside the material which is given by

$$B_M = \mu_0 n I_M$$

The total magnetic field inside a magnetic material is the sum of the external magnetising field and the additional magnetic field produced due to magnetisation of the material and is called magnetic induction $\vec{B}$. The ***magnetic induction*** *may also be defined as the total number of magnetic lines of force crossing per unit area normally through a material.* Thus the SI unit of magnetic induction is *tesla* (T) or weber metre^{-2} (Wbm^{-2}) which is equivalent to Nm^{-1}A^{-1} or JA^{-1}m^{-2}.

3. **Magnetising field intensity.** The ability of magnetising field to magnetise a material medium is expressed by a vector $\vec{H}$, called *magnetising field intensity* or *magnetic intensity. Its magnitude may be defined as the number of ampere-turns* (nI) *flowing round the unit length of the solenoid required to produce the given magnetising field.* Thus

$$H = nI$$

$$\therefore \quad B_0 = \mu_0 nI = \mu_0 H \quad \text{or} \quad H = \frac{B_0}{\mu_0}$$

The dimensions of magnetic intensity are [L^{-1}A]. Its SI unit is *ampere metre*$^{-1}$ (Am^{-1}) which is equivalent to Nm^{-2}T^{-1} or Jm^{-1}Wb^{-1}.

4. **Intensity of magnetisation.** When a magnetic material is placed in a magnetising field, it gets magnetised. *The magnetic moment developed per unit volume of a material when placed in a magnetising field is called intensity of magnetisation or simply* ***magnetisation***. Thus

$$\vec{M} = \frac{\vec{m}}{V}$$

If I_M is the surface magnetisation current set up in a solenoid of cross-sectional area A and having n turns per unit length, then magnetic moment developed per unit length of the solenoid is $nI_M A$. Therefore, magnetic moment developed per unit volume or the magnetisation $\vec{M}$ is given by

$$M = \frac{m}{V} = \frac{n I_M A}{A} = n I_M$$

Hence $\quad B_M = \mu_0 n I_M = \mu_0 M$

Again, consider a bar of magnetic material having cross-sectional area a and length $2l$. Its volume is

$$V = a \times 2l$$

Suppose the bar develops pole strength q_m when placed in a magnetising field, then its magnetic moment,

$$m = q_m \times 2l$$

$$\therefore \quad M = \frac{m}{V} = \frac{q_m \times 2l}{a \times 2l} = \frac{q_m}{a}$$

Hence *intensity of magnetisation may also be defined as the pole strength developed per unit cross-sectional area of a material.*

As the total magnetic field or the magnetic induction $\vec{B}$ inside a magnetic material is the resultant of the magnetising field $\vec{B}_0$ and the field $\vec{B}_M$ produced due to the magnetisation of the material, therefore,

$$B = B_0 + B_M = \mu_0 H + \mu_0 M$$

or $\quad B = \mu_0 (H + M)$

Clearly, both H and M have the same units, namely Am^{-1}.

5. Magnetic Permeability. Permeability is the measure of the extent to which a material can be penetrated or permeated by a magnetic field. *The magnetic permeability of a material may be defined as the ratio of its magnetic induction B to the magnetic intensity H.*

$$\mu = \frac{B}{H}$$

Clearly, SI unit of μ

$$= \frac{\text{tesla}}{\text{ampere metre}^{-1}}$$

$$= \text{tesla metre ampere}^{-1} \quad \text{or} \quad \text{TmA}^{-1}$$

$\therefore$ Dimensions of $\mu = [\text{MLT}^{-2}\text{A}^{-2}]$.

6. Relative permeability. Permeability of various magnetic substances can be compared with one another in terms of relative permeability μ_r. *It is defined as the ratio of the permeability of the medium to the permeability of free space.* Thus,

$$\mu_r = \frac{\mu}{\mu_0}$$

For vacuum $\mu_r = 1$, for air it is 1.0000004 and for iron, the value of μ_r may exceed 1000.

7. Magnetic susceptibility. Magnetic susceptibility measures the ability of a substance to take up magnetisation when placed in a magnetic field. *It is defined as the ratio of the intensity of magnetisation M to the magnetising field intensity H.* It is denoted by χ_m. Thus,

$$\chi_m = \frac{M}{H}$$

As magnetic susceptibility is the ratio of two quantities having the same units (Am^{-1}), so it has no units.

8. Relation between magnetic permeability and magnetic susceptibility. If a linear magnetic material, subjected to the action of a magnetising field intensity H, develops magnetisation M and magnetic induction B ; then

$$B = \mu_0(H + M)$$

But $$B = \mu H$$

$$\therefore \quad \mu H = \mu_0(H + M)$$

or $$\mu = \mu_0\left(1 + \frac{M}{H}\right) \quad \text{or} \quad \mu = \mu_0(1 + \chi_m)$$

or $$\mu_0\mu_r = \mu_0(1 + \chi_m) \quad \text{or} \quad \mu_r = 1 + \chi_m.$$

5.24 CLASSIFICATION OF MAGNETIC MATERIALS

30. *How are materials classified on the basis of their behaviour in a magnetic field ? Give examples of each type.*

Classification of magnetic materials. On the basis of their behaviour in external magnetic fields, Faraday classified the various substances into *three* categories :

1. **Diamagnetic substances.** *Diamagnetic substances are those which develop feeble magnetisation in the opposite direction of the magnetising field. Such substances are feebly repelled by magnets and tend to move from stronger to weaker parts of a magnetic field.*

Examples. Bismuth, copper, lead, zinc, tin, gold, silicon, nitrogen (at STP), water, sodium chloride, etc.

2. **Paramagnetic substances.** *Paramagnetic substances are those which develop feeble magnetisation in the direction of the magnetising field. Such substances are feebly attracted by magnets and tend to move from weaker to stronger parts of a magnetic field.*

Examples. Manganese, aluminium, chromium, platinum, sodium, copper chloride, oxygen (at STP), etc.

3. **Ferromagnetic substances.** *Ferromagnetic substances are those which develop strong magnetisation in the direction of the magnetising field. They are strongly attracted by magnets and tend to move from weaker to stronger parts of a magnetic field.*

Examples. Iron, cobalt, nickel, gadolinium and alloys like alnico.

5.25 ORIGIN OF DIAMAGNETISM

31. *Explain the origin of diamagnetism. Why are the diamagnetic substances repelled by magnets ?*

Origin of diamagnetism. In atoms of some materials like Bi, Cu, Pb, the magnetic moments due to different electrons cancel out. In such atoms, electrons occur in pairs with one of them revolving clockwise

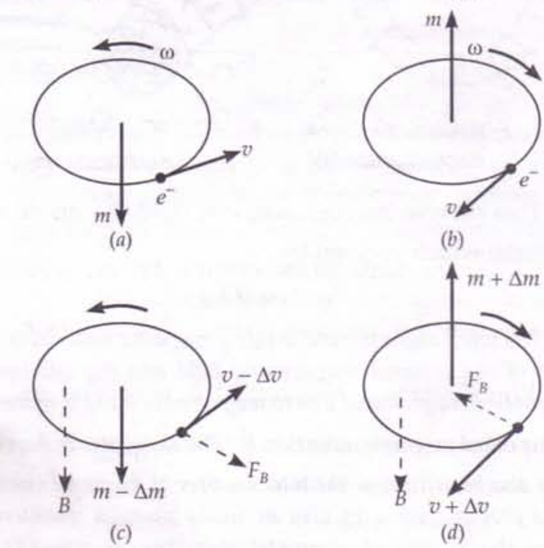


Fig. 5.42 An electron orbiting in an atom produces a moment.

and other anticlockwise around the nucleus. Net magnetic moment of an atom is zero, as shown in Fig. 5.42(*a*) and (*b*).

When such an atom is placed in a magnetic field $\vec{B}$, the speed of revolution of one electron increases and that of other decreases. The magnetic moment of the former electron increases to $\vec{m} + \Delta\vec{m}$ and that of the latter electron decreases to $\vec{m} - \Delta\vec{m}$. So each electron pair gains a net magnetic moment $2\Delta\vec{m}$ which is proportional to the field $\vec{B}$ but points in its *opposite* direction as shown in Figs. 5.42(*c*) and (*d*). A sufficient magnetic moment is induced in the diamagnetic sample in the opposite direction of $\vec{B}$. This sample moves from stronger to the weaker parts of the field $\vec{B}$, *i.e.*, a diamagnetic substance is repelled by a magnet. The behaviour of diamagnetic materials is independent of temperature.

5.26 ORIGIN OF PARAMAGNETISM

32. *Explain the origin of paramagnetism. State Curie's law of magnetism.*

Origin of paramagnetism. According to *Langevin*, the atoms or molecules of a paramagnetic material possess a *permanent magnetic moment* either due to the presence of some unpaired electron or due to the non-cancellation of the spins of two electrons because of some special reason. In the absence of an external magnetic field, the atomic dipoles are randomly oriented due to their ceaseless random motion, as shown in Fig. 5.43(*a*). There is no net magnetisation.

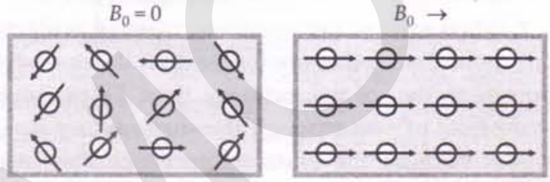


Fig. 5.43 (*a*) Randomly distributed atomic dipoles in a paramagnetic material in the absence of magnetic field.
(*b*) Alignment of dipoles in the presence of magnetic field.

When a strong enough field $\vec{B_0}$ is applied and the temperature is low enough, the field $\vec{B_0}$ tends to align the atomic dipoles in its own direction, producing a weak magnetic moment in the direction of $\vec{B_0}$. The material tends to move from a weak field region to a strong field region. This is paramagnetism.

At very high magnetic fields or at very low temperatures, the magnetisation approaches its maximum value when all the atomic dipole moments get aligned. This is called the *saturation magnetisation value* M_s.

Curie's law. From experiments, it is found that the *intensity of magnetisation* (*M*) of a paramagnetic material is

(*i*) *directly proportional to the magnetising field intensity H*, because the latter tends to align the atomic dipole moments.

(*ii*) *inversely proportional to the absolute temperature T*, because the latter tends to oppose the alignment of the atomic dipole moments.

Therefore at low H/T values, we have

$$M \propto \frac{H}{T}$$

or
$$M = C \cdot \frac{H}{T}$$

or
$$\frac{M}{H} = \frac{C}{T} \quad \text{or} \quad \chi_m = \frac{C}{T}$$

Here C is curie constant and χ_m is the susceptibility of the material. The above relation is called *Curie's law*. This law *states that far away from saturation, the susceptibility of a paramagnetic material is inversely proportional to the absolute temperature.*

Figure 5.44 shows the variation of intensity of magnetisation M as a function of H/T. Beyond the saturation value M_s, Curie law is not valid.

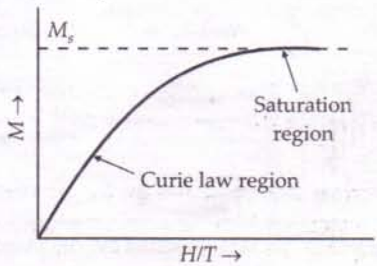


Fig. 5.44 Magnetisation M as a function of H/T.

5.27 ORIGIN OF FERROMAGNETISM : DOMAIN THEORY

33. *Describe ferromagnetism on the basis of domain theory. How does Curie's law get modified for ferromagnetic substances ?*

Origin of ferromagnetism. *Weiss* explained ferromagnetism on the basis of his domain theory. In materials like Fe, Ni, Co, the individual atoms are associated with large magnetic moments. The magnetic moments of neighbouring atoms interact with each other and align themselves spontaneously in a common direction over macroscopic regions called *domains*. Each domain has a typical size of about 1 mm and contains about 10^{11} atoms. So each domain possesses a strong magnetic moment. In the absence of any external magnetic field, these domains are randomly distributed so that the net magnetic moment is zero.

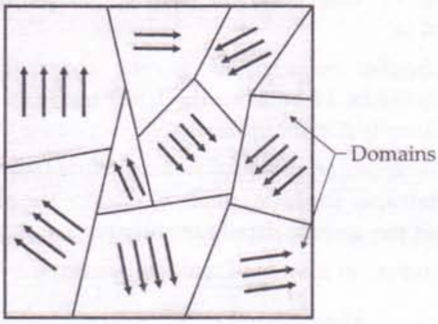
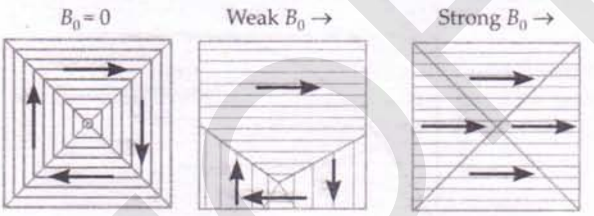


Fig. 45 Randomly oriented domains in a ferromagnetic substance.

When a ferromagnetic material is placed in a magnetic field, all the domains align themselves along the direction of the field leading to the strong magnetisation of the material along the direction of the field. That is why the ferromagnetic substances are strongly attracted by magnets. The alignment of domains may occur in either of the following two ways :

1. ***By displacement of the boundaries of domains.*** When the external field B_0 is weak, the domains aligned in the direction of B_0 grow in size while those oppositely directed decrease in size, as shown in Fig. 5.46(*b*).



(*a*) Unmagnetised sample (*b*) Magnetisation by growing of domains (*c*) Magnetisation by rotation of domains

Fig. 5.46 Magnetisation of a ferromagnetic sample.

2. ***By rotation of domains.*** When the external field B_0 is strong, the domains rotate till their magnetic moments get aligned in the direction of B_0 as shown in Fig. 5.46(*c*).

Modified Curie's law for ferromagnetic substances. When a ferromagnetic sample is heated, its magnetisation decreases due to the increase in the randomisation of its domains. At a sufficiently high temperature, the domain structure disintegrates and the ferromagnetic substance becomes paramagnetic. *The temperature at which a ferromagnetic substance becomes paramagnetic is called **Curie temperature** or **Curie point** T_C.*

Above the curie point *i.e.*, in the paramagnetic phase, the susceptibility varies with temperature as

$$\chi_m = \frac{C'}{T - T_C} \qquad (T > T_C)$$

where C' is a constant. This is *modified Curie's law* for a ferromagnetic material above the Curie temperature. It is also known as ***Curie-Weiss law***. This law states that *the susceptibility of a ferromagnetic substance above its Curie temperature is inversely proportional to the excess of temperature above the Curie temperature.*

Table 5.2 Curie Temperatures of some Ferromagnetic Materials

Material	T_C (K)
Cobalt	1394
Iron	1043
Fe_2O_3	893
Nickel	631
Gadolinium	317

5.28 PROPERTIES OF DIAMAGNETIC SUBSTANCES

34. *Describe some of the important properties of diamagnetic substances.*

Properties of diamagnetic substances :

1. When placed in an external magnetic field, a diamagnetic substance develops feeble magnisation in the opposite direction of the applied field.

2. When a rod of a diamagnetic material is placed in a magnetic field, poles are induced on it in a direction opposite to that of the inducing field. So the lines of force prefer to pass through the surrounding air than to pass through the material itself *i.e.*, the lines of force get expelled or repelled, as shown in Fig. 5.47. Consequently, the magnetic induction B inside the material becomes less than the magnetising field, $B_0 = \mu_0 H$. The reduction is very small, about 1 part in 10^5.

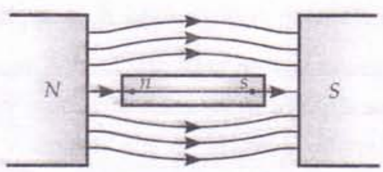


Fig. 5.47 Reduction of lines of force in a diamagnetic rod.

3. *When placed in a non-uniform magnetic field, a diamagnetic substance moves from stronger to the weaker parts of the field.*

When a watch glass containing a diamagnetic liquid is placed over two closely lying (3 – 4 mm apart) pole pieces of a magnet, the liquid is found to move towards the poles causing a depression in the middle [Fig 5.48(*a*)]. This indicates that the field is stronger in the middle than that near the poles. Now if the poles are moved apart sufficiently, the magnetic field at the middle becomes weaker than that near the poles. Consequently, the liquid accumulates in the middle and thins out near the poles [Fig. 5.48(*b*)].

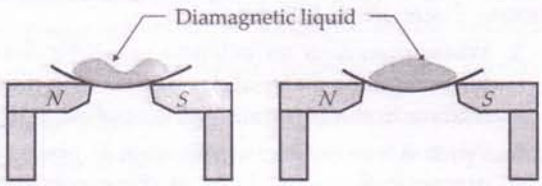


Fig. 5.48 Effect of non-uniform magnetic field on a diamagnetic liquid when (*a*) poles are quite close to each other, (*b*) poles are sufficiently apart.

4. When a rod of a diamagnetic material is suspended freely in a uniform magnetic field, it aligns itself perpendicular to the magnetising field (Fig. 5.49).

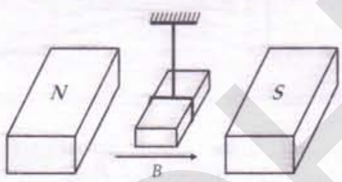


Fig. 5.49 A freely suspended diamagnetic rod in a uniform field.

5. As a diamagnetic substance develops a weak magnetisation in the opposite direction of the magnetising field, the susceptibility ($\chi_m = M/H$) of diamagnetic materials is small and negative. For bismuth, $\chi_m = -0.00015$.

6. The relative permeability μ_r $(=1+\chi_m)$ is positive but less than 1 for a diamagnetic material.

7. The susceptibility of diamagnetic substances is independent of the magnetising field and the temperature, as shown in Fig. 5.50.

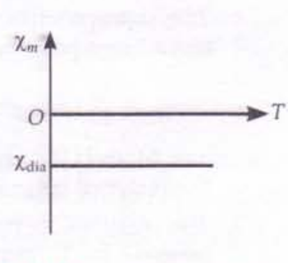


Fig. 5.50 χ_m-T graph for diamagnetic material.

8. The magnetisation of a diamagnetic substance lasts so long as the magnetising field is applied.

35. *Briefly describe diamagnetism in superconducting metals.*

Diamagnetism in superconducting metals. When a metal is cooled to a temperature below its critical temperature in a magnetic field, it attains both superconductivity and perfect diamagnetism. The magnetic lines of force get completely expelled from it and it repels a magnet. For this material, $\chi = -1$ and $\mu_r = 0$. *This phenomenon of diamagnetism in superconductors is called* ***Meissner effect***. This effect forms the basis for running magnetically levitated superfast trains.

5.29 PROPERTIES OF PARAMAGNETIC SUBSTANCES

36. *Describe some of the important properties of paramagnetic substances.*

Properties of paramagnetic substances :

1. When placed in an external magnetic field, a paramagnetic substance develops feeble magnetisation in the direction of the applied field.
2. When a rod of paramagnetic material is placed in a magnetic field, the lines of force prefer to pass through it than through the surrounding air *i.e.*, the lines of force get slightly more concentrated inside the material, as shown in Fig. 5.51. The magnetic induction B becomes slightly greater than the magnetising field, $B_0 = \mu_0 H$. The increase is very small, about 1 part in 10^5.

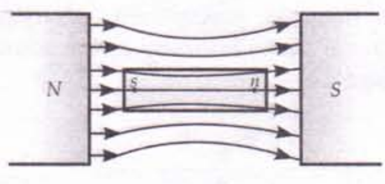


Fig. 5.51 Slightly cocentrated lines of force in a paramagnetic rod.

3. *When placed in a non-uniform magnetic field, a paramagnetic substance moves from weaker to the stronger parts of the field.*

   When a watch glass containing a paramagnetic liquid is placed over two closely lying pole pieces of a magnet, the liquid accumulates and elevates in the middle and thins out near the poles [Fig. 5.52(*a*)]. This is because the field in the centre is the strongest. When the poles are moved apart, the field at the poles becomes stronger than that at the centre and the liquid moves towards the poles [Fig. 5.52(*b*)].

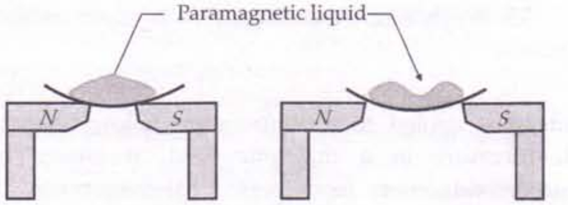


Fig. 5.52 Effect of magnetic field on a paramagnetic liquid when (*a*) poles are quite close to each other, (*b*) poles are farther apart.

4. When a rod of paramagnetic material is suspended freely in a uniform magnetic field, it aligns itself parallel to the magnetising field (Fig. 5.53).

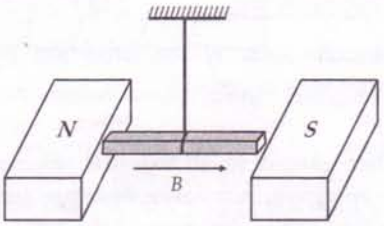


Fig. 5.53 A freely suspended paramagnetic rod in a uniform magnetic field.

5. A paramagnetic material develops small magnetisation in the direction of the magnetising field, so its susceptibility has small but positive value. For aluminium, $\chi = 1.8 \times 10^{-6}$.
6. The relative permeability $(\mu_r = 1 + \chi_m)$ for a paramagnetic material has a value slightly greater than 1.
7. The magnetic susceptibility of a paramagnetic material varies inversely as the absolute temperature, *i.e.*

$$\chi_m \propto \frac{1}{T}$$

or $$\chi_m = \frac{C}{T},$$

where C is a constant called the *Curie constant* and this equation is known as *Curie's law*.

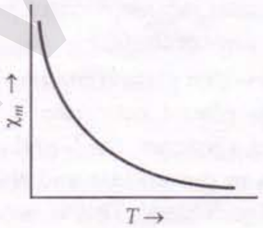


Fig. 5.54 χ_m-T graph for a paramagnetic material.

8. For a given temperature, the intensity of magnetisation is proportional to the magnetising field, so the susceptibility and permeability do not show any variation with the field $\vec{B_0}$.
9. As soon as the magnetising field is removed, a paramagnetic substance loses its magnetism.

5.30 PROPERTIES OF FERROMAGNETIC SUBSTANCES

37. *Describe some of the important properties of ferromagnetic substances.*

Properties of ferromagnetic substances. Ferromagnetic substances exhibit properties similar to those of paramagnetic substances but in a highly dominant manner. These are as follows :

1. When placed in an external magnetic field, a ferromagnetic material develops strong magnetisation in the direction of the applied field.
2. When a ferromagnetic substance is placed in a magnetic field, the lines of force concentrate greatly into the material so that the magnetic induction B becomes much more than the magnetising field B_0.

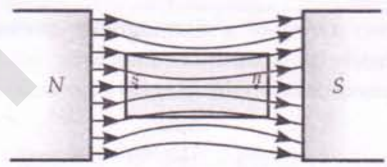


Fig. 5.55 Highly concentrated lines of force in a ferromagnetic rod.

3. When a ferromagnetic substance is placed in non-uniform magnetic field, it moves from weaker to the stronger parts of the field.
4. When a rod of a ferromagnetic material is suspended freely in a uniform magnetic field, it quickly aligns itself parallel to the magnetic field.
5. The intensity of magnetisation M is proportional to the magnetising field intensity H for its smaller values. For moderate values of H, M increases rapidly and then finally attains constant value for large H. This indicates the attainment of the saturation stage of magnetisation.
6. The susceptibility of a ferromagnetic material has a large positive value. This is because

$$\chi_m = \frac{M}{H}$$

and $M >> H$ for a ferromagnetic material. It is of the order of several thousands.
7. The relative permeability $(\mu_r = 1 + \chi_m)$ of a ferromagnetic material has a large positive value. It is of the order of several thousands. For iron, $\mu_r = 1000$.

8. The susceptibility of ferromagnetic material decreases with temperature in accordance with Curie-Weiss law :

$$\chi_m = \frac{C}{T - T_C} \qquad (T > T_C)$$

9. At a certain temperature called the *Curie point*, the susceptibility suddenly falls and the ferromagnetic substance becomes paramagnetic.
10. The magnetisation developed depends not only on the value of magnetising field but also on the past magnetic and mechanical history of the material.
11. A ferromagnetic substance retains magnetism even after the magnetising field is removed.

For Your Knowledge

- In the presence of an external magnetic field, magnetic moments are induced in all materials. Hence diamagnetism is universal. But paramagnetism and ferromagnetism are much stronger than diamagnetism, so it is difficult to detect diamagnetism in para- and ferro-magnetic substances.
- Magnetic materials are broadly classified as diamagnetic, paramagnetic and ferromagnetic. However, there exist some other types of magnetic materials with mysterious properties. These include ferrimagnetic, anti-ferromagnetic, spin glass, etc.
- A very small variation in the value of χ_m may lead to an altogether different magnetic behaviour : diamagnetic *vs.* paramagnetic. For diamagnetic materials, $\chi_m \simeq -10^{-5}$ where $\chi_m = +10^{-5}$ for paramagnetic materials.

Table 5.3 Comparative study of the properties of dia-, para- and ferromagnetic substances

	Property	Diamagnetic substances	Paramagnetic substances	Ferromagnetic substances
1.	*Effect of magnets*	They are feebly repelled by magnets.	They are feebly attracted by magnets.	They are strongly attracted by magnets.
2.	*In external magnetic field*	Acquire feeble magnetisation in the opposite direction of the magnetising field.	Acquire feeble magnetisation in the direction of the magnetising field.	Acquire strong magnetisation in the direction of the magnetising field.
3.	*In a non-uniform magnetic field*	Tend to move slowly from stronger to weaker parts of the field.	Tend to move slowly from weaker to stronger parts of the field.	Tend to move quickly from weaker to stronger parts of the field.
4.	*In a uniform magnetic field*	A freely suspended diamagnetic rod aligns itself perpendicular to the field.	A freely suspended paramagnetic rod aligns itself parallel to the field.	A freely suspended ferromagnetic rod aligns itself parallel to the field.
5.	*Susceptibility value* (χ_m)	Susceptibility is small and negative. $-1 \le \chi_m < 0$	Susceptibility is small and positive. $0 < \chi_m < \varepsilon$, where ε is a small number	Susceptibility is very large and positive. $\chi_m > 1000$
6.	*Relative permeability value* (μ_r)	Slightly less than 1 $0 \le \mu_r < 1$	Slightly greater than 1 $1 < \mu_r < 1 + \varepsilon$	Of the order of thousands $\mu_r > 1000$
7.	*Permeability value* (μ)	$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$
8.	*Effect of temperature*	Susceptibility is independent of temperature.	Susceptibility varies inversely as temperature : $\chi_m \propto \frac{1}{T}$.	Susceptibility decreases with temperature in a complex manner. $\chi_m \propto \frac{1}{T - T_C} \; (T > T_C)$
9.	*Removal of magnetising field*	Magnetisation lasts as long as the magnetising field is applied.	As soon as the magnetising field is removed, magnetisation is lost.	Magnetisation is retained even after the magnetising field is removed.
10.	*Variation of M with H*	*M* changes linearly with *H*.	*M* changes linearly with *H* and attains saturation at low temperature and in very strong fields.	*M* changes with *H* non-linearly and ultimately attains saturation.
11.	*Hysteresis effect*	*B*-vector shows no hysteresis.	*B*-vector shows no hysteresis.	*B*-vector shows hysteresis.
12.	*Physical state of the material*	Solid, liquid or gas.	Solid, liquid or gas.	Normally solids only.
13.	*Examples*	Bi, Cu, Pb, Si, N_2 (at STP), H_2O, NaCl	Al, Na, Ca, O_2 (at STP), $CuCl_2$	Fe, Ni, Co, Gd, Fe_2O_3, Alnico.

5.31 HYSTERESIS

38. *Explain the phenomenon of hysteresis in magnetic materials. What is the significance of the area of hysteresis loop ? State the practical importance of hysteresis loops.*

Hysteresis. When a ferromagnetic sample is placed in a magnetising field, the sample gets magnetised by induction. As the magnetising field intensity H varies, the magnetic induction B does not vary linearly with H, *i.e.*, the permeability $\mu (= B/H)$ is not constant but varies with H. In fact, it also depends on the past history of the sample.

Figure 5.56 shows the variation of magnetic induction B with magnetising field intensity H. Point O represents the initial unmagnetised state of a ferromagnetic sample. As the magnetising field intensity H increases, the magnetic induction B first gradually increases and then attains a constant value. In other words, the magnetic induction B *saturates* at a certain value $+H_{max}$.

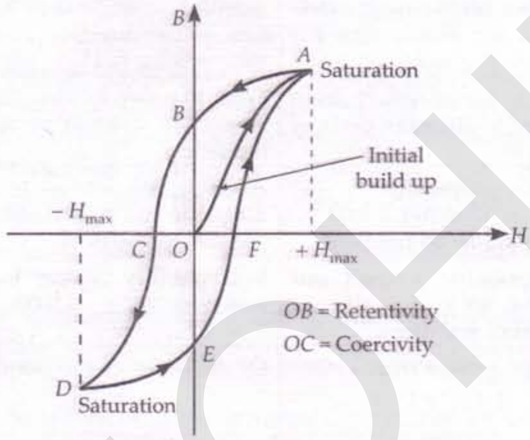


Fig. 5.56 Hysteresis loop for a ferromagnetic sample.

Now if the magnetising field intensity H is gradually decreased to zero, B decreases but along a new path AB. It is found that the magnetic induction B does not become zero even when the magnetising field H is zero, *i.e.*, the sample is not demagnetised even when the magnetising field has been removed. *The magnetic induction (= OB) left behind in the sample after the magnetising field has been removed is called* ***residual magnetism*** *or* ***retentivity*** or ***remanence***.

To reduce the magnetism to zero, the field H is gradually increased in the *reverse* direction, the induction B decreases and becomes zero at a value of $H = OC$. *The value of reverse magnetising field intensity H required for the residual magnetism of a sample to become zero is called* ***coercivity*** *of the sample.*

On further increasing H in the reverse direction to a value $-H_{max}$, we reach the saturation point D located symmetrically to point A. Now if H is decreased gradually, the point A is reached after going through the path $DEFA$.

The closed curve $ABCDEFA$ which represents a *cycle of magnetisation* of a ferromagnetic sample is called its ***hysteresis loop***. Throughout the cycle, the magnetic field B lags behind the magnetising field intensity H, *i.e.*, the value of B when H is decreasing is always more than when H is increasing. *The phenomenon of the lagging of magnetic induction behind the magnetising field is called* **hysteresis**. In fact, the word hysteresis originates from a Greek word meaning '*delayed*'.

Significance of the area of hysteresis loop. The product $BH = B\left(\frac{B}{\mu}\right) = \frac{B^2}{\mu_0 \mu_r}$, has the dimensions of energy per unit volume. Hence the area within the B-H loop represents *the energy dissipated per unit volume* in the material when it is carried through a cycle of magnetisation. The *source* is the source of emf used in magnetising the material and the *sink* is the hysteretic heat loss in the magnetic material.

Practical importance of hysteresis loops. A study of hysteresis loop provides us information about retentivity, coercivity and hysteresis loss of a magnetic material. This helps in proper selection of materials for designing cores of transformers and electromagnets and in making permanent magnets.

39. *Distinguish between soft and hard ferromagnetic materials. Draw their hysteresis loops. Give examples of each type.*

Types of ferromagnetic materials. Ferromagnetic materials can be divided into two categories :

1. ***Soft ferromagnetic materials or soft ferromagnets.*** *These are the ferromagnetic materials in which the magnetisation disappears on the removal of the external magnetising field.* Such materials have narrow hysteresis loop, as shown in Fig. 5.57(*a*). Consequently, they

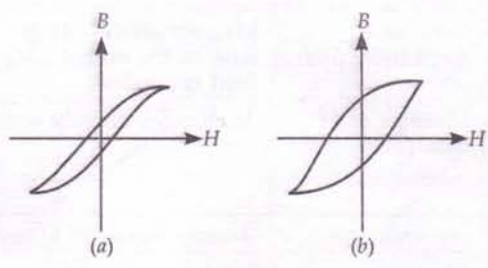


Fig. 5.57 Magnetic hysteresis loop for (*a*) soft, (*b*) hard ferromagnetic material.

have low retentivity, low coercivity, and low hysteresis loss. But they have high relative magnetic permeability. They are used as cores of solenoids and transformers. Examples. Soft iron, mu metal, stalloy, etc.

2. ***Hard ferromagnetic materials or hard ferromagnets.*** *These are the ferromagnetic materials which retain magnetisation even after the removal of the external magnetising field.* Such materials have wide hysteresis loop, as shown in Fig. 5.57(*b*). Consequently, they have high retentivity, high coercivity and large hysteresis loss. They are used for making permanent magnets.

Examples. Steel, alnico, lodestone, ticonal, etc.

Examples based on Magnetic Properties of Materials

Formulae Used

1. Intensity of magnetisation, $M = \frac{m}{V}$
2. $\mu = \frac{B}{H}$
3. $\mu_r = \frac{\mu}{\mu_0}$
4. $\chi_m = \frac{M}{H}$
5. $\chi_m = \frac{C}{T}$ [Curie's law]
6. $B = \mu_0 (H + M)$
7. $\mu_r = 1 + \chi_m$.

Units Used

Magnetising field intensity H is in Am^{-1}, field B in tesla, magnetisation M in Am^{-1}, permeability μ in TmA^{-1} or Hm^{-1}, susceptibility χ_m and relative permeability μ_r have no units.

Example 39. *A magnet of magnetic moment 2.5 Am^2 weighs 66 g. If the density of the material of the magnet is 7500 kgm^{-3}, find the intensity of magnetisation.*

Solution. Volume,

$$V = \frac{\text{Mass}}{\text{Density}} = \frac{66 \times 10^{-3} \text{ kg}}{7500 \text{ kgm}^{-3}} = \frac{66 \times 10^{-5}}{75} \text{ m}^3$$

Magnetisation,

$$M = \frac{m}{V} = \frac{2.5}{\frac{66 \times 10^{-5}}{75}} = \frac{2.5 \times 75 \times 10^5}{66}$$

$$= \mathbf{2.84 \times 10^5 \text{ Am}^{-1}}.$$

Example 40. *Obtain the earth's magnetisation. Assume that the earth's field can be approximated by a gaint bar magnet of magnetic moment 8.0×10^{22} Am^2. The earth's radius is 6400 km.* [NCERT]

Solution. Here magnetic moment,

$$m = 8.0 \times 10^{22} \text{ Am}^2$$

Radius of the earth, $R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$

Magnetisation,

$$M = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3} = \frac{8.0 \times 10^{22} \times 3}{4 \times 3.14 \times (6.4 \times 10^6)^3}$$

$$= \mathbf{72.9 \text{ Am}^{-1}}.$$

Example 41. *A domain in ferromagnetic iron is in the form of a cube of side length 1 μm. Estimate the number of iron atoms in the domain and the maximum possible dipole moment and magnetisation of the domain. The molecular mass of iron is 55 g / mole and its density is 7.9 g / cm^3. Assume that each iron atom has a dipole moment of 9.27×10^{-24} Am^2.* [NCERT]

Solution. Each side of cubic domain,

$$l = 1 \,\mu\text{m} = 10^{-6} \text{ m}$$

Volume of the domain,

$$V = l^3 = (10^{-6} \text{ m})^3 = 10^{-18} \text{m}^3 = 10^{-12} \text{ cm}^3$$

Mass of domain = Volume × density

$$= 10^{-12} \text{ cm}^3 \times 7.9 \text{ g cm}^{-3}$$

$$= 7.9 \times 10^{-12} \text{g}$$

Number of atoms in 55 g iron

$$= 1 \text{ mole} = 6.023 \times 10^{23}$$

∴ Number of atoms in 7.9×10^{-12} g iron

$$= \frac{6.023 \times 10^{23} \times 7.9 \times 10^{-12}}{55}$$

or $$N = \mathbf{8.65 \times 10^{10} \text{ atoms}}.$$

Dipole moment of each iron atom,

$$m = 9.27 \times 10^{-24} \text{Am}^2$$

The dipole moment of the domain will be maximum when all its atomic dipoles get perfectly aligned. Its value will be

$$m_{max} = mN = 9.27 \times 10^{-24} \times 8.65 \times 10^{10}$$

$$= \mathbf{8.0 \times 10^{-13} \text{ Am}^2}.$$

The maximum possible magnetisation of the domain,

$$M = \frac{m_{max}}{V} = \frac{8.0 \times 10^{-13} \text{Am}^2}{10^{-18} \text{m}^3}$$

$$= \mathbf{8.0 \times 10^5 \text{ Am}^{-1}}.$$

Example 42. *A magnetising field of 1500 A / m produces a magnetic flux of 2.4×10^{-5} weber in a bar of iron of cross-section 0.5 cm^2. Calculate permeability and susceptibility of the iron-bar used.* [CBSE OD 08]

Solution. Here $H = 1500 \text{ Am}^{-1}$, $\phi = 2.4 \times 10^{-5}$ Wb, $A = 0.5 \times 10^{-4} \text{m}^2$

Magnetic induction,

$$B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.5 \times 10^{-4}} = 0.48 \text{ Wbm}^{-2}$$

Permeability,

$$\mu = \frac{B}{H} = \frac{0.48}{1500} = \mathbf{3.2 \times 10^{-4}\ TmA^{-1}}$$

As $\mu = \mu_0 (1 + \chi_m)$

∴ Susceptibility,

$$\chi_m = \frac{\mu}{\mu_0} - 1 = \frac{3.2 \times 10^{-4}}{4 \times 3.14 \times 10^{-7}} - 1$$

$$= 254.77 - 1 = \mathbf{253.77}.$$

Example 43. *Assume that each iron atom has a permanent magnetic moment equal to 2 Bohr magnetons (1 Bohr magneton* $= 9.27 \times 10^{-24}\ Am^2$*). The number density of atoms in iron is* $8.52 \times 10^{28}\ m^{-3}$*. (i) Find the maximum magnetisation M in a long iron bar. (ii) Find the maximum magnetic induction B in the bar.*

Solution. (*i*) Number of atoms per unit volume,

$$n = 8.52 \times 10^{28}\ \text{m}^{-3}$$

Magnetic moment of each iron atom

$$= 2\,\mu_B = 2 \times 9.27 \times 10^{-24}\ \text{Am}^2$$

As magnetisation M is the magnetic moment per unit volume, so the maximum value of magnetisation is

$$M_{max} = n \times 2\mu_B$$

(when all the dipoles get aligned)

$$= 8.52 \times 10^{28} \times 2 \times 9.27 \times 10^{-24}$$

$$= \mathbf{1.58 \times 10^6\ Am^{-1}}.$$

(*ii*) Magnetic induction, $B = \mu_0 (H + M)$

As no magnetising field is applied, so $H = 0$. Hence

$$B = \mu_0 M = 4\pi \times 10^{-7} \times 1.58 \times 10^6 = \mathbf{1.985\ T}.$$

Example 44. *A solenoid of 500 turns/m is carrying a current of 3 A. Its core is made of iron which has a relative permeability of 5000. Determine the magnitudes of the magnetic intensity, magnetisation and the magnetic field inside the core.* [NCERT]

Solution. Here $n = 500$ turns/m, $I = 3\,A$, $\mu_r = 5000$

Magnetic intensity,

$$H = nI = 500\ \text{m}^{-1} \times 3\ \text{A} = \mathbf{1500\ Am^{-1}}.$$

As $\mu_r = 1 + \chi_m$

∴ $\chi_m = \mu_r - 1 = 5000 - 1 = 4999 \simeq 5000$

Also, $\mu_r = \frac{\mu}{\mu_0} = 5000$ or $\mu = 5000\,\mu_0$

Magnetisation,

$$M = \chi_m H = 5000 \times 1500$$

$$= \mathbf{7.5 \times 10^6\ Am^{-1}}.$$

Magnetic field inside the core,

$$B = \mu H = 5000\,\mu_0 H$$

$$= 5000 \times 4\pi \times 10^{-7} \times 1500 = 3\pi \simeq \mathbf{9.4\ T}.$$

Example 45. *The core of a toroid having 3000 turns has inner and outer radii of 11 cm and 12 cm respectively. The magnetic field in the core for a current of 0.70 A is 2.5 T. What is the relative permeability of the core ?*

Solution. The magnetic field in the empty space enclosed by a toroid is given by

$$B = \mu_0 nI$$

where n is the number of turns per unit length and I is the current. If the space is filled by a core of permeability μ, then

$$B = \mu nI$$

Here $B = 2.5$ T, $I = 0.70$ A

Mean radius,

$$r = \frac{11 + 12}{2}\ \text{cm} = 11.5\ \text{cm} = 11.5 \times 10^{-2}\ \text{m}$$

$$\therefore\quad n = \frac{3000}{2\pi r} = \frac{3000}{2 \times 3.14 \times 11.5 \times 10^{-2}}$$

Hence

$$\mu = \frac{B}{nI} = \frac{2.5 \times 2 \times 3.14 \times 11.5 \times 10^{-2}}{3000 \times 0.70}$$

$$= 8.6 \times 10^{-4}\ \text{Tm A}^{-1}$$

Relative permeability,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{8.6 \times 10^{-4}}{4\pi \times 10^{-7}} = \mathbf{684.4}.$$

Example 46. *An iron rod of volume* $10^{-4}\,m^3$ *and relative permeability 1000 is placed inside a long solenoid wound with 5 turns per cm. If a current of 0.5 A is passed through the solenoid, find the magnetic moment of the rod.*

Solution. The relation between the magnetic induction B, magnetising field intensity H and the magnetisation M is given by

$$B = \mu_0 (H + M)$$

$$\therefore\quad M = \frac{B}{\mu_0} - H = \frac{\mu H}{\mu_0} - H \qquad [\because B = \mu H]$$

$$= \mu_r H - H = (\mu_r - 1) H \qquad \left[\mu_r = \frac{\mu}{\mu_0}\right]$$

But for a long solenoid, we have

$$H = nI$$

where n is the number of turns per metre.

$$\therefore\quad M = (\mu_r - 1) nI$$

Here $\mu_r = 1000$, $I = 0.5$ A

$$n = \frac{5}{0.01}\ \text{turns/m} = 500\ \text{turns/m}$$

$$\therefore\ M = (1000 - 1) \times 500 \times 0.5 = 2.5 \times 10^5\ \text{Am}^{-1}$$

Magnetic moment,

$$m = M \times V = 2.5 \times 10^5 \times 10^{-4}\ \text{Am}^2 = \mathbf{25\ Am^2}.$$

Example 47. *The hysteresis loss for a specimen of iron weighing 12 kg is equivalent to 300 Jm^{-3} $cycle^{-1}$. Find the loss of energy per hour at 50 cycle s^{-1}. Density of iron is 7500 kg m^{-3}.*

Solution. Let Q be the energy dissipated per unit volume per hysteresis cycle in the given sample. Then the total energy lost by the volume V of the sample in time t will be

$$W = Q \times V \times \nu \times t$$

where ν is the number of hysteresis cycles per second.

Here $Q = 300\ \text{Jm}^{-3}\ \text{cycle}^{-1}$, $\nu = 50\ \text{cycle s}^{-1}$, $t = 1\ \text{h} = 3600\ \text{s}$

Volume, $V = \dfrac{\text{Mass}}{\text{Density}} = \dfrac{12}{7500}\ \text{m}^3$

∴ Hysteresis loss,

$$W = 300 \times \frac{12}{7500} \times 50 \times 3600\ \text{J} = \mathbf{86400\ J.}$$

Example 48. *The coercivity of a certain permanent magnet is $4.0 \times 10^4\ Am^{-1}$. This magnet is placed inside a solenoid 15 cm long and having 600 turns and a current is passed in the solenoid to demangnetise it completely. Find the current.*

Solution. The coercivity of $4 \times 10^4\ \text{Am}^{-1}$ of the permanent magnet implies that a magnetic intensity $H = 4 \times 10^4\ \text{Am}^{-1}$ is required to be applied in opposite direction to demagnetise the magnet.

Here $n = \dfrac{600}{15\ \text{cm}} = \dfrac{600}{15 \times 10^{-2}\ \text{m}} = 4000\ \text{turns / m}$

As $H = nI$

∴ Current, $I = \dfrac{H}{n} = \dfrac{4 \times 10^4}{4000} = \mathbf{10\ A.}$

Problems For Practice

1. A bar magnet made of steel has a magnetic moment of 2.5 Am^2 and a mass of 6.6 g. If the density of steel is $7.9 \times 10^3\ \text{kg m}^{-3}$, find the intensity of magnetisation of the magnet. (Ans. $3.0 \times 10^6\ \text{Am}^{-1}$)
2. The maximum value of permeability of μ-metal (77% Ni, 16% Fe, 5% Cu, 2% Cr) is 0.126 TmA^{-1}. Find the maximum relative permeability and susceptibility. (Ans. $\mu_r = 1.0 \times 10^5$, $\chi \simeq 1.0 \times 10^5$)
3. Find the percent increase in the magnetic field B when the space within a current-carrying toroid is filled with aluminium. The susceptibility of aluminium is 2.1×10^{-5}. (Ans. 2.1×10^{-3})
4. The susceptibility of magnesium at 300 K is 1.2×10^{-5}. At what temperature will the susceptibility increase to 1.8×10^{-5}. (Ans. 200 K)
5. An iron rod of 0.2 cm^2 cross-sectional area is subjected to a magnetising field of 1200 Am^{-1}. The susceptibility of iron is 599. Find the permeability and the magnetic flux produced. (Ans. $7.536 \times 10^{-4}\ \text{Tm A}^{-1}$, 1.81×10^{-5} Wb)
6. An iron rod 0.2 m long, 10 mm in diameter and of permeability 1000 is placed inside a long solenoid wound with 300 turns per metre. If a current of 0.5 ampere is passed through the solenoid, find the magnetic moment of the rod. (Ans. 0.2325 Am^{-1})
7. An iron ring of mean circumferential length 30 cm and cross-section 1 cm^2 is wound uniformly with 300 turns of wire. When a current of 0.032 A flows in the windings ; the flux in the ring is 2×10^{-6} Wb. Find the flux density in the ring, magnetising field intensity and relative permeability of iron. (Ans. $2 \times 10^{-2}\ \text{Wb m}^{-2}$, 32 A turns m^{-1}, 500)
8. An iron ring having 500 turns of wire and a mean diameter of 12 cm carries a current of 0.3 A. The relative permeability of iron is 600. What is the magnetic flux density in the core ? What is the magnetisation field intensity ? What part of the flux density is due to the electronic loop currents in the core ? (Ans. 0.3 Wbm^{-1}, 397. 9 A turns m^{-1}, 0.2995 Wbm^{-2})

HINTS

1. Proceed as in Example 39 on page 5.37.
2. Maximum relative permeability,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.126}{4\pi \times 10^{-7}} = \mathbf{1.0 \times 10^5}$$

Maximum susceptibility,

$$\chi = \mu_r - 1 \simeq \mathbf{1.0 \times 10^5}.$$

3. In the absence of aluminium,

$$B_0 = \mu_0 H$$

In the presence of aluminium

$$B = \mu H = \mu_0 (1 + \chi) H$$

Increase in field $= B - B_0 = \mu_0 \chi H$

Percent increase $= \dfrac{B - B_0}{B_0} \times 100 = \dfrac{\mu_0 \chi H}{\mu_0 H} \times 100$

$$= \chi \times 100 = \mathbf{2.1 \times 10^{-3}}.$$

4. $\dfrac{\chi_2}{\chi_1} = \dfrac{T_1}{T_2}$ ∴ $T_2 = \dfrac{\chi_1}{\chi_2} \cdot T_1 = \dfrac{1.2 \times 10^{-5} \times 300}{1.8 \times 10^{-5}} = \mathbf{200\ K.}$

5. Here $A = 0.2\ \text{cm}^2 = 0.2 \times 10^{-4}\ \text{m}^2$, $H = 1200\ \text{Am}^{-1}$, $\chi_m = 599$

Permeability, $\mu = \mu_0 (1 + \chi_m)$

$$= 4\pi \times 10^{-7} \times (1 + 599) = \mathbf{7.536 \times 10^{-4}\ TmA^{-1}}$$

Magnetic induction,

$$B = \mu H = 7.536 \times 10^{-4} \times 1200 = 0.904 \text{ T}$$

Magnetic flux,

$$\phi = BA = 0.904 \times 0.2 \times 10^{-4} = \mathbf{1.81 \times 10^{-5} \ Wb.}$$

6. Proceed as in Example 46 on page 5.38.

7. Here $l = 30 \text{ cm} = 0.30 \text{ m}, \ A = 1 \text{cm}^2 = 10^{-4} \text{m}^2,$

$$N = 300, \ I = 0.032 \text{ A}, \ \phi = 2 \times 10^{-6} \text{ Wb},$$

$$n = \frac{N}{l} = \frac{300}{0.30} = 1000 \text{ m}^{-1}$$

Magnetic flux density,

$$B = \frac{\phi}{A} = \frac{2 \times 10^{-6}}{10^{-4}} = \mathbf{2 \times 10^{-2} \ Wb \ m^{-2}}$$

Magnetising field intensity,

$$H = nI = 1000 \times 0.032 = \mathbf{32 \ A \ turns \ m^{-1}}$$

Permeability,

$$\mu = \frac{B}{H} = \frac{2 \times 10^{-2}}{32} = 6.25 \times 10^{-4} \text{ T m A}^{-1}.$$

Relative permeability, $\mu_r = \frac{\mu}{\mu_0} = \frac{6.25 \times 10^{-4}}{4\pi \times 10^{-7}} = \mathbf{500.}$

8. Here $N = 500, \ D = 12 \text{ cm} = 0.12 \text{ m}, \ I = 0.3 \text{ A},$ $\mu_r = 600$

Magnetising field intensity,

$$H = nI = \frac{N}{l} \cdot I = \frac{N}{\pi D} \cdot I$$

$$= \frac{500 \times 0.3}{\pi \times 0.12} = \mathbf{397.9 \ A \ turns \ m^{-1}}$$

Magnetic flux density, $B = \mu H = \mu_r \mu_0 H$

$$= 600 \times 4\pi \times 10^{-7} \times 397.9 = \mathbf{0.3 \ Wbm^{-1}}$$

Flux density due to electronic current loop is

$$\mu_0 M = B - \mu_0 H = 0.3 - 4\pi \times 10^{-7} \times 397.9$$

$$= 0.3 - 0.0005 = \mathbf{0.2995 \ Wbm^{-2}}.$$

5.32 PERMANENT MAGNETS AND ELECTROMAGNETS

40. *Give a comparison of the magnetic properties of soft iron and steel.*

Comparison of the magnetic properties of soft iron and steel. Fig. 5.58 shows the hysteresis loops for soft iron and steel.

A study of these $B - H$ loops reveals the following information :

1. ***Permeability.*** For a given H, B is more for soft iron than steel. So soft iron has a greater permeability ($\mu = B/H$) than steel.

2. ***Susceptibility.*** As permeability of soft iron is greater than steel, so soft iron has a greater susceptibility ($\chi_m = \mu_r - 1$) than steel.

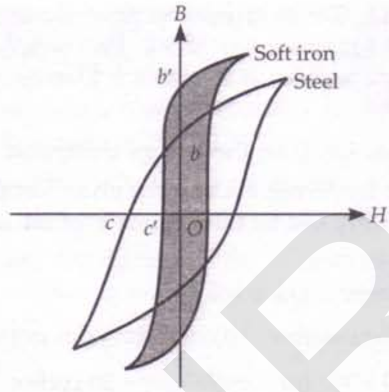


Fig. 5.58 Hysteresis loops of soft iron and steel.

3. ***Retentivity.*** The retentivity of soft iron (Ob') is greater than the retentivity (Ob) of steel.

4. ***Coercivity.*** The coercivity of soft iron (Oc') is less than the coercivity (Oc) of steel.

5. ***Hysteresis loss.*** As the area of the hysteresis loop of soft iron is much smaller than that of steel, so the hysteresis loss per unit volume per cycle is less for soft iron than for steel.

We can summarise the above properties as follows :

1. Permeability 2. Susceptibility 3. Retentivity	are greater for soft iron than for steel
4. Coercivity 5. Hysteresis loss	are less for soft iron than for steel

41. *How will you select materials for making permanent magnets, electromagnets and cores of transformers ?*

Selection of magnetic materials. The choice of magnetic materials for making permanent magnets, electromagnets and cores of transformers is decided from the hysteresis loop of the material.

A. Permanent magnets. The material used for making permanent magnets must have the following characteristics :

1. *High retentivity* so that it produces a strong magnetic field.

2. *High coercivity* so that its magnetisation is not destroyed by stray magnetic fields, temperature variations or minor mechanical damage.

3. *High permeability.*

Inspite of its slightly smaller retentivity than soft iron, steel is favoured for making permanent magnets. Steel has much higher coercivity than soft iron. The magnetisation of steel is not easily destroyed by stray fields. Once magnetised under a strong field, it retains magnetisation for a long duration. Other suitable materials for making permanent magnets are :

Cobalt steel	52% Fe, 36% Co, 7% W, 3.5% Cr, 0.5% Mn, 0.7% C
Carbon steel	98% Fe, 0.86% C, 0.9% Mn
Alnico	55% Fe, 10% Al, 17% Ni, 12% Co, 6% Cu
Ticonal	42% Co, 26.5 Fe, 14% Ni, 8% Al, 6.5 Ti, 3% Cu

B. Electromagnets. The material used for making cores of electromagnets must have the following characteristics :

1. *High initial permeability* so that magnetisation is large even for a small magnetising field.

2. *Low retentivity* so that the magnetisation is lost as the magnetising current is switched off.

So soft iron is more suitable than steel for cores of electromagnets.

C. Transformer cores. The material used for making cores of transformers must have the following characteristics :

1. *High initial permeability* so that the magnetic flux is large even for low magnetising fields.

2. *Low hysteresis loss* as the materials are subjected to alternating magnetising fields of high frequency.

3. *Low resistivity* to reduce losses due to eddy currents.

Soft iron is preferred for making transformer cores and telephone diaphragms.

42. *Mention three methods for making permanent magnets.*

Methods for making permanent magnets. A hard ferromagnetic material like steel can be converted into a permanent magnet by any of the following methods :

1. By holding the steel rod in north-south direction and hammering it repeatedly.
2. Hold a steel rod and stroke it with one end of a bar magnet a number of times, always in the same sense to make a permanent magnet.
3. The most efficient way of making a permanent magnet is to place a steel rod in a solenoid and pass a strong current. The rod gets magnetised due to the magnetic field of the solenoid.

43. *Briefly explain how an electromagnet is formed. State some uses of electromagnets.*

Electromagnet. As shown in Fig. 5.59, take a soft iron rod and wind a large number of turns of insulated copper wire over it. When we pass a current through the solenoid, a magnetic field is set up in the space within the solenoid. The high permeability of soft iron increases the field one thousand times. The end of the solenoid at which the current in the solenoid seems to flow anticlockwise acts as N-pole and other one as S-pole. When the current in the solenoid is switched off, the soft iron rod loses its magnetism almost completely due to its low retentivity.

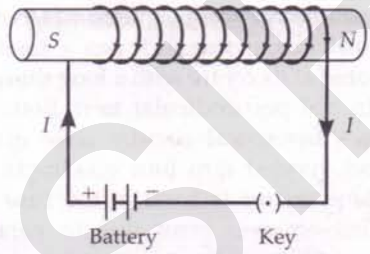


Fig. 5.59 An electromagnet.

Uses of electromagnets :

1. Electromagnets are used in electric bells, loudspeakers and telephone diaphragms.
2. Large electromagnets are used in cranes to lift heavy machinery, and bulk quantities of iron and steel.
3. In hospitals, electromagnets are used to remove iron or steel bullets from the human body.

5.33 TANGENT GALVANOMETER*

44. *State tangent law of magnetism.*

Tangent law. This law states that if a freely suspended small magnet is acted upon by two uniform mutually perpendicular magnetic fields B_1 and B_2 simultaneously, then the magnet comes to rest in such

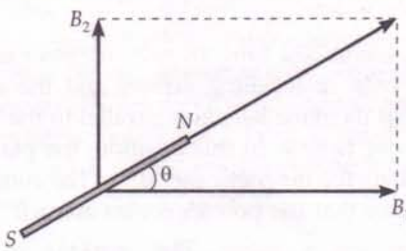


Fig. 5.60 Tangent law.

a position that the tangent of the angle θ that the magnet makes with B_1 is equal to the ratio B_2 / B_1 of the two fields. That is,

$$\tan\theta = \frac{B_2}{B_1}$$

or

$$B_2 = B_1 \tan\theta$$

45. *Describe the principle, construction, theory and working of a tangent galvanometer.*

Tangent galvanometer. It is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.

Construction. It consists of a circular frame of non-magnetic material mounted on a horizontal turn table. Three coils having 2, 50 and 500 turns of insulated copper wire are wound over it. The ends of the coils are connected to three base terminals.

A compass box of non-magnetic material is fitted at the centre of circular frame. It has a small magnetic needle pivoted at its centre with a long thin aluminium pointer attached perpendicular to it. Both are free to move over a horizontal circular scale graduated in degrees and divided into four quadrants of $0° - 90°$ each. A plane mirror is fixed at the base below the pointer. This removes error due to parallax while reading the position of the pointer.

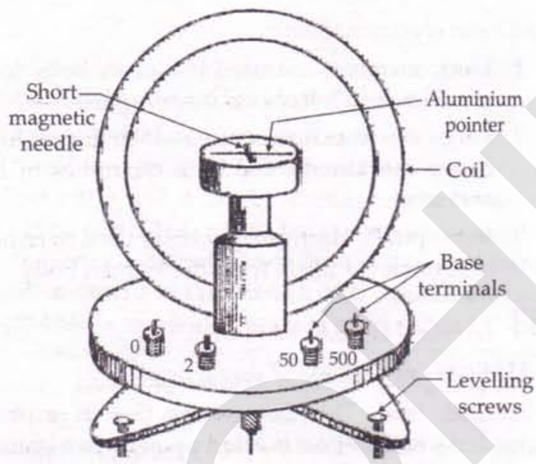


Fig. 5.61 Tangent galvanometer.

Adjustments The tangent galvanometer is levelled with the help of levelling screws and the coil is so rotated that its plane becomes parallel to the length of the magnetic needle. In this position, the plane of the coil lies along the magnetic meridian. The compass box is rotated, so that the pointer comes along $0° - 0°$ line.

Theory and working. The working of tangent galvanometer is based on tangent law. When no current is passed through the coil, the magnetic needle is influenced only by B_H of earth's magnetic field. When a current I is passed, there is a magnetic field B along the axis of coil perpendicular to B_H, as shown in Fig. 5.62. The magnetic needle is influenced by two perpendicular magnetic fields and it comes to rest at an angle θ with B_H such that

$$B = B_H \tan\theta$$

This relation is known as *Tangent law.*

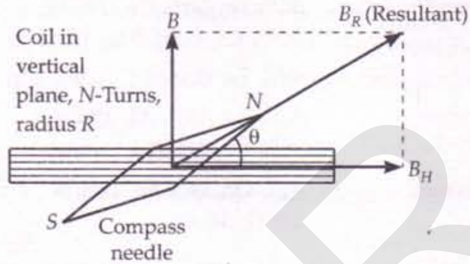


Fig. 5.62

Now magnetic field at centre of coil is

$$B = \frac{\mu_0 NI}{2R}$$

where N and R are number of turns and radius of the coil.

$$\therefore \quad \frac{\mu_0 N I}{2R} = B_H \tan\theta$$

$$I = \frac{2R B_H \tan\theta}{\mu_0 N} = K \tan\theta$$

where $K = \frac{2R B_H}{\mu_0 N}$ is a constant for the tangent galvanometer and is called its ***reduction factor***.

5.34 OSCILLATIONS OF A FREELY SUSPENDED MAGNET

46. *Show that the oscillations of a freely suspended magnet in a uniform magnetic field are simple harmonic. Hence deduce an expression for its time period.*

Oscillations of a freely suspended magnet in a magnetic field. In the position of equilibrium, the magnetic dipole lies along $\vec{B}$. When it is slightly rotated from this position and released, it begins to vibrate about the field direction under the restoring torque,

$$\tau = -mB\sin\theta$$

The negative sign indicates that the direction of toque τ is such so as to decrease θ.

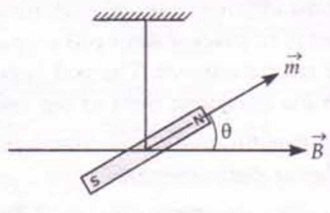


Fig. 5.63

For small angular displacement θ, $\sin\theta \approx \theta$

$\therefore \quad \tau = -mB\theta$

If I is the moment of inertia of the magnet, then the deflecting torque on the magnet is

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

In the equilibrium condition,

Deflecting torque = Restoring torque

$$\therefore \quad I\frac{d^2\theta}{dt^2} = -mB\theta$$

or
$$\frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta = -\omega^2\theta$$

i.e., angular acceleration $\frac{d^2\theta}{dt^2} \propto$ angular displacement θ.

Hence the oscillations of a freely suspended magnetic dipole in a uniform magnetic field are simple harmonic. The time period of oscillation is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mB}}.$$

Example 49. *In Fig. 5.64, a magnetic needle is free to oscillate in a uniform magnetic field. The magnetic needle has magnetic moment 6.7 Am² and moment of inertia $I = 7.5 \times 10^{-6}$ km m². It performs 10 complete oscillations in 6.70 s. What is the magnitude field ?* [NCERT]

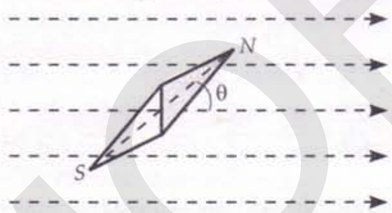


Fig. 5.64

Solution. Here $T = \frac{6.70 \text{ s}}{10} = 0.67$ s, $m = 6.7 \text{ Am}^2$,

$I = 7.5 \times 10^{-6} \text{ kg m}^2$

As
$$T = 2\pi\sqrt{\frac{I}{mB}}$$

or
$$T^2 = 4\pi^2\frac{I}{mB}$$

The magnitude of the magnetic field is

$$B = \frac{4\pi^2 I}{mT^2} = \frac{4 \times 9.87 \times 7.5 \times 10^{-6}}{6.7 \times (0.67)^2}$$

$$= \mathbf{9.8 \times 10^{-5}\ T.}$$

5.35 VIBRATION MAGNETOMETER*

47. *Describe the principle, construction and working of a vibration magnetometer. Mention its uses.*

Vibration magnetometer. It is *an instrument used to compare the magnetic moments of two magnets or to determine the horizontal component of earth's magnetic field at a place.*

Principle. When a magnet suspended freely in a uniform magnetic field (like the one due to the earth), is displaced from its equilibrium position, it begins to vibrate simple harmonically about the direction of the field. The period of vibration is given by

$$T = 2\pi\sqrt{\frac{I}{mB_H}}$$

where, m = magnetic moment of the magnet,

B_H = horizontal component of earth's magnetic field,

I = the moment of inertia of the magnet about an axis of rotation through its centre of mass

and
$$I = \text{Mass} \times \frac{l^2 + b^2}{12}$$

Here l is the length and b the breadth of the magnet.

Construction. It consists of a short magnet enclosed in a wooden box provided with glass windows. The box has a narrow tube fixed on its top at the middle. The magnet is suspended horizontally in a light *brass* stirrup by a silk thread which passes centrally down the tube and is provided with a torsion-head at the top of the tube. The glass box protects the magnet from air current and its top has two slits through which vibrations of the magnet can be observed. A plane mirror strip with a reference line on it is placed *lengthwise* at the base of the instrument just below the slits.

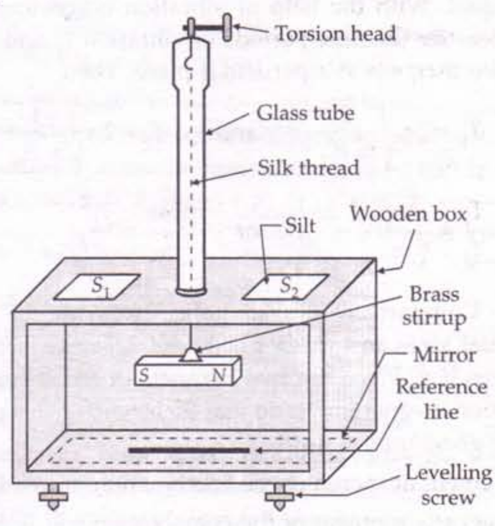


Fig. 5.65 Vibration magnetometer.

Adjustments

(i) Place a compass needle on the reference line. Rotate the box till the line becomes parallel to the compass needle. This sets the magnetometer in the north-south direction.

(ii) To ensure that there is no twist in the thread, place a brass bar of the same size as that of the magnet in the stirrup and allow the stirrup to come to rest. Adjust the torsion head so that the brass bar is in the north-south direction.

(iii) Replace the brass bar by a small magnet with its N-pole pointing geographic north. Bring a powerful magnet near the box and remove it. The suspended magnet starts oscillating. Note its period of vibration.

48. *Explain the various uses of a vibration magnetometer.*

Uses of a vibration magnetometer :

1. **Measurement of magnetic moment of a magnet.** Set the vibration magnetometer in the north-south direction. Place the bar magnet in its stirrup. Measure the time period of the bar magnet.

As $$T = 2\pi\sqrt{\frac{I}{mB_H}}$$

$$\therefore \quad T^2 = 4\pi^2 \frac{I}{mB_H} \quad \text{or} \quad m = \frac{4\pi^2 I}{B_H \cdot T^2}$$

The moment of inertia I can be determined from the geometry of the magnet. Knowing B_H, magnetic moment m can be determined.

2. **Comparison of magnetic moments of two magnets of same size and same mass.** For the two bar magnets of same size and mass, the moments of inertia are equal. With the help of vibration magnetometer, we measure the time periods of vibration T_1 and T_2 of the two magnets at a particular place. Then

$$T_1 = 2\pi\sqrt{\frac{I}{m_1 B_H}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{I}{m_2 B_H}}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{m_2}{m_1}} \quad \text{or} \quad \frac{m_2}{m_1} = \frac{T_1^2}{T_2^2}.$$

3. **Comparison of magnetic moments of two unequal sizes and masses** *(sum and difference method).* *Sum position.* Place the two magnets in the stirrup of vibration magnetometer so that their north poles point in the geographical north.

Moment of inertia of the combination $= I_1 + I_2$

Magnetic moment of the combination $= m_1 + m_2$

Determine the period of vibration of the combination. Let it be T_1. Then

$$T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 + m_2)B_H}} \quad ...(1)$$

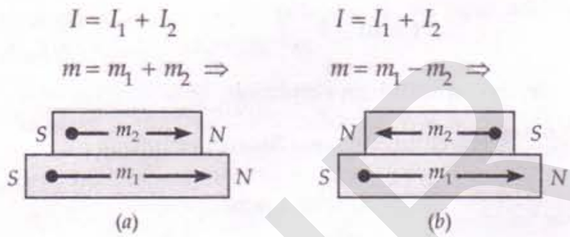


Fig. 5.66 (a) Sum position. (b) Difference position.

Difference position. Now place the two magnets in the stirrup of the vibration magnetometer with their opposite poles in the same direction.

Moment of inertia of the combination

$$= I_1 + I_2$$

Magnetic moment of the combination

$$= m_1 - m_2$$

Let period of vibration of the combination

$$= T_2$$

Then $$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 - m_2)B_H}} \quad ...(2)$$

Dividing (1) by (2), we get

$$\frac{T_1}{T_2} = \sqrt{\frac{m_1 - m_2}{m_1 + m_2}}$$

or $$\frac{T_1^2}{T_2^2} = \frac{m_1 - m_2}{m_1 + m_2}$$

By componendo and dividendo, we get

$$\frac{m_2}{m_1} = \frac{T_2^2 - T_1^2}{T_2^2 + T_1^2}.$$

4. **Comparison of horizontal components of earth's magnetic field.** With the help of a vibration magnetometer, we measure the periods of vibration T and T' of the same magnet at the two given places. Let B_H and B'_H be the horizontal components of earth's magnetic field at these places.

Then $$T = 2\pi\sqrt{\frac{I}{mB_H}} \quad \text{and} \quad T' = 2\pi\sqrt{\frac{I}{mB'_H}}$$

$$\therefore \quad \frac{T}{T'} = \sqrt{\frac{B'_H}{B_H}} \quad \text{or} \quad \frac{B_H}{B'_H} = \frac{T'^2}{T^2}$$

GUIDELINES TO NCERT EXERCISES

5.1. *Answer the following questions regarding earth's magnetism :*

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

(b) The angle of dip at a location in southern India is about 18°. Would you expect a greater or smaller dip angle in Britain ?

[CBSE OD 95C]

(c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground ?

(d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole ? [CBSE D 95C]

(e) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8×10^{22} JT^{-1} located at its centre. Check the order of magnitude of this number in some way.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all ?

Ans. (*a*) The three independent quantities used to specify the earth's magnetic field are

(*i*) magnetic declination, (*ii*) angle of dip, and

(*iii*) horizontal component of earth's magnetic field.

(*b*) Britain is closer to the magnetic north pole. So the angle of dip is greater in Britain than that in India. It is about 70° in Britain.

(*c*) Magnetic lines of force of earth's magnetism will seem to come out of the ground at Melbourne in Australia because this region lies in the southern hemisphere of the earth where the earth's magnetic north pole lies.

(*d*) Earth's magnetic field is exactly vertical at the poles and so the horizontal component of earth's field is zero which makes the compass needle point in any direction at the geomagnetic north or south pole.

(*e*) Magnetic field $\vec{B}$ at an equatorial point of the earth's magnetic dipole is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

Now $m = 8 \times 10^{22}\ \text{JT}^{-1},\ r = 6.4 \times 10^{6}\ \text{m}$

$$\therefore \quad B = 10^{-7} \times \frac{8 \times 10^{22}}{(6.4 \times 10^{6})^{3}}\ \text{T}$$

$$= 0.3 \times 10^{-4}\ \text{T} = 0.3\ \text{G}$$

which is of the same order of magnitude as that of the observed field on the earth.

(*f*) The earth's field is only approximately a dipole field. Local N-S poles may arise due to the different deposits of magnetised minerals.

5.2. *Answer the following questions :*

(a) The earth's magnetic field varies from point to point in space. Does it also change with time ? If so, on what time scale does it change appreciably ?

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why ?

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents ?

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past ?

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion ?

(f) Interstellar space has an extremely weak magnetic field of the order of 10^{-12} T. Can such a weak field be of any significant consequence ? Explain.

Ans. (*a*) Yes, it does change with time. Time scale for appreciable change is roughly a few hundred years. But even on a much smaller scale of a few years, its variations are not completely negligible.

(*b*) The temperature of earth's core is very high, so iron exists as molten iron which, being at a temperature higher than Curie point is not ferromagnetic.

(*c*) Radioactivity may be one of the possible sources for the charged current in the outer conducting regions of the earth's core which are thought to be responsible for earth's magnetism.

(*d*) Earth's magnetic field gets recorded weakly in certain rocks during their solidification. An analysis of these rocks may reveal the history of earth's magnetism.

(*e*) At large distances, the field gets modified due to the field of ions in motion (in the earth's ionosphere). The field of these ions, in turn, is sensitive to extraterrestrial disturbances such as the solar wind.

(*f*) When a charged particle moves in a magnetic field, it gets deflected along a circular path of radius,

$$R = \frac{mv}{eB} \qquad \left[\because\ evB = \frac{mv^2}{R}\right]$$

A weak field of 10^{-12} T bends the charged particle in a circle of very large radius. Over a small distance, we may not notice the deflection but over very large interstellar distance, the deflection is quite noticeable.

5.3. *A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} J. What is the magnetic moment of the magnet ?*

Ans. Here $\theta = 30°,\ B = 0.25\ \text{T},\ \tau = 4.5 \times 10^{-2}\ \text{J},\ m = ?$

As $\tau = mB \sin\theta$,

$$\therefore \quad m = \frac{\tau}{B \sin\theta} = \frac{4.5 \times 10^{-2}}{0.25 \times \sin 30°} = \mathbf{0.36\ JT^{-1}}.$$

5.4. *A short bar magnet of magnetic moment $m = 0.32\ \text{JT}^{-1}$ is placed in a uniform external magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientations would correspond to its (i) stable and (ii) unstable equilibrium ? What is the potential energy of the magnet in each case ?*

Ans. Here $m = 0.32\ \text{JT}^{-1},\ B = 0.15\ \text{T}$

(*i*) The bar will be in *stable equilibrium* when its magnetic moment $\vec{m}$ is parallel to $\vec{B}\,(\theta = 0°)$. Its potential energy is then minimum and is given by

$$U_{min} = -mB\cos 0° = -0.32 \times 0.15 \times 1$$
$$= \mathbf{-4.8 \times 10^{-2}\ J}.$$

(*ii*) The bar will be in unstable equilibrium when its magnetic moment $\vec{m}$ is *antiparallel* to $\vec{B}\,(\theta = 180°)$. Its potential energy is then maximum and is given by

$$U_{max} = -mB\cos 180° = -0.32 \times 0.15 \times (-1)$$
$$= \mathbf{+4.8 \times 10^{-2}\ J}.$$

5.5. *A closely wound solenoid of 800 turns and area of cross-section $2.5 \times 10^{-4}\ m^2$ carries a current of 3.0 A. Explain in what sense does the solenoid act like a bar magnet. What is its associated magnetic moment ?*

Ans. Here $N = 800,\ A = 2.5 \times 10^{-4}\ \text{m}^2,\ I = 3.0\ \text{A}$

$$m = NIA = 800 \times 3 \times 2.5 \times 10^{-4} = \mathbf{0.60\ J\,T^{-1}}$$

The magnetic field of a solenoid has the same pattern as that of a bar magnet. It acts along the axis of the solenoid. Its direction is determined by the sense of flow of current.

5.6. *If the solenoid in Exercise 5.5 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of the torque on the solenoid when its axis makes an angle of 30° with the direction of the magnetic field ?*

Ans. Here $m = 0.60\ \text{JT}^{-1},\ B = 0.25\ \text{T},\ \theta = 30°$

$$\tau = mB\sin\theta = 0.60 \times 0.25 \times \sin 30° = \mathbf{7.5 \times 10^{-2}\ J}.$$

5.7. *A bar magnet of magnetic moment 1.5 JT^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.*

(a) What is the amount of work required to turn the magnet so as to align its magnetic moment (i) normal to the field direction (ii) opposite to the field direction ?

(b) What is the torque on the magnet in cases, (i) and (ii) ?

Ans. Here $m = 1.5\ \text{JT}^{-1}$, $B = 0.22\ \text{T}$

(i) Given $\theta_1 = 0°$, $\theta = 90°$

$$\therefore \quad W = -mB(\cos\theta_2 - \cos\theta_1)$$
$$= -1.5 \times 0.22 (\cos 90° - \cos 0°)$$
$$= -0.33 \times (0 - 1) = \mathbf{+0.33\ J}$$

Torque, $\tau = mB \sin 90° = 1.5 \times 0.22 \times 1 = \mathbf{0.33\ Nm.}$

(ii) Given $\theta_1 = 0°$, $\theta_2 = 180°$

$$W = -1.5 \times 0.22 \times \cos(180° - \cos 0°)$$
$$= -0.33 \times (-1 - 1) = \mathbf{0.66\ J}$$

Torque, $\tau = mB \sin 180° = 1.5 \times 0.22 \times 0 = \mathbf{0.}$

5.8. *A closely wound solenoid of 2000 turns and area of cross-section $1.6 \times 10^{-4} m^2$, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.*

(a) *What is the magnetic moment associated with the solenoid ?*

(b) *What are the force and torque on the solenoid if a uniform horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30° with the axis of the solenoid ?* [CBSE OD 15C]

Ans. Here $N = 2000$, $A = 1.6 \times 10^{-4}\ \text{m}^2$, $I = 4.0$ A

(a) Magnetic moment of solenoid of turns N, area of cross-section A and carrying current I is

$$m = NIA = 2000 \times 4.0 \times 1.6 \times 10^{-4}\ \text{Am}^2$$
$$= \mathbf{1.28\ Am^2}$$

This magnetic moment acts along the axis of the solenoid in a direction related to the sense of current via the right-hand screw rule.

(b) Net force experienced by the magnetic dipole in the uniform magnetic field

$$= 0$$

The magnitude of the torque τ exerted by the magnetic field $\vec{B}$ on the solenoid is given by

$$\tau = mB \sin\theta = 1.28 \times 7.5 \times 10^{-2} \times \sin 30°$$
$$= \mathbf{0.048\ Nm}$$

This torque tends to align the axis of the solenoid (*i.e.*, its magnetic moment vector $\vec{m}$) along the field $\vec{B}$.

5.9. *A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of $2.0\ s^{-1}$. What is the moment of inertia of the coil about its axis of rotation ?*

Ans. Here $N = 16$, $r = 10$ cm $= 0.10$ m, $I = 0.75$ A

$$B = 5.0 \times 10^{-2}\ \text{T},\ \nu = 2.0\ \text{s}^{-1}$$

Magnetic moment of the coil is $m = NIA = NI \cdot \pi r^2$

Frequency of oscillation, $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{mB}{I}}$

$\therefore$ Moment of inertia is

$$I = \frac{mB}{4\pi^2\nu^2} = \frac{NI\pi r^2 \cdot B}{4\pi^2\nu^2}$$
$$= \frac{16 \times 0.75 \times (0.1)^2 \times 5 \times 10^{-2}}{4 \times 3.14 \times 4}$$
$$= \mathbf{1.2 \times 10^{-4}\ kg\ m^2.}$$

5.10. *A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22° with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine magnitude of earth's magnetic field at the place.*

Ans. Here $\delta = 22°$, $B_H = 0.35$ G, $B = ?$

$$B = \frac{B_H}{\cos\delta} = \frac{0.35\ \text{G}}{\cos 22°} = \frac{0.35\ \text{G}}{0.9272} = \mathbf{0.38\ G.}$$

5.11. *At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of the magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of earth's field at the location.*

Ans. Here $B_H = 0.16$ G, $\delta = 60°$

$$\therefore \quad B = \frac{B_H}{\cos\delta} = \frac{0.16}{\cos 60°} = \frac{0.16}{0.5} = \mathbf{0.32\ G}$$

Thus the earth's magnetic field has a magnitude of 0.32 G and lies in a vertical plane 12° west of the geographic meridian making an angle of 60° (upwards) with the horizontal (magnetic south to magnetic north) direction.

5.12. *A short bar magnet has a magnetic moment of 0.48 JT^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (i) the axis (ii) the equatorial line of the magnet.*

Ans. Here $m = 0.48\ \text{JT}^{-1}$, $r = 10$ cm $= 0.10$ m

(i) For a short magnet, $B_{axial} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2m}{r^3}$

$$= \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 0.48}{(0.10)^3} = 0.96 \times 10^{-4}\ \text{T} = \mathbf{0.96\ G.}$$

This field acts along *S-N* direction.

(ii) For a short magnet, $B_{equa} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{m}{r^3}$

$$= \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{0.48}{(0.10)^3} = 0.48 \times 10^{-4}\ \text{T} = \mathbf{0.48\ G.}$$

This field acts along *N-S* direction.

5.13. *A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null-points (i.e., 14 cm) from the centre of the magnet ?*

Ans. As the null points lie on the axis of the magnet, therefore

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = B_H$$

Magnetic field of the magnet on its normal bisector at the same distance will be

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = \frac{B_H}{2} = \frac{0.36}{2} = 0.18\,\text{G}$$

∴ Total magnetic field at the required point on the normal bisector is

$$B_{equa} + B_H = 0.18 + 0.36 = \mathbf{0.54\ G}.$$

5.14. *If the bar magnet in Exercise 5.13 is turned around by 180°, where will the new null points be located ?*

Ans. When the magnet is turned around by 180°, its south pole will lie in the geographical south direction. The null points will now lie on the equatorial line of the magnet, say at distance x from the centre of the magnet. Then

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{x^3} = B_H$$

But $\quad B_H = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$ [From Exercise 5.13]

$$\therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{m}{x^3} \quad \text{or} \quad \frac{2}{r^3} = \frac{1}{x^3}$$

or $\quad x^3 = \frac{r^3}{2}$

$$\therefore \quad x = \frac{r}{2^{1/3}} = \frac{14\,\text{cm}}{1.26} = \mathbf{11.1\ cm.}$$

5.15. *A short bar magnet of magnetic moment $5.25 \times 10^{-2}\ JT^{-1}$ is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet on (i) its normal bisector (ii) its axis, is the resultant field inclined at 45° with the earth's field. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of magnet in comparison to the distance involved.*

Ans. Here $m = 5.25 \times 10^{-2}\ \text{JT}^{-1}$,

$$B_0 = 0.42\,\text{G} = 0.42 \times 10^{-4}\ \text{T}$$

(*i*) Figure 5.95(*a*) shows a point P on the normal bisector of a magnet where the resultant field is inclined at 45° with the earth's field B_0. As point P is an equatorial point, therefore, the resultant field must be such that

$$\frac{B_{equa}}{B_0} = \tan 45°$$

or $\quad B_{equa} = B \tan 45° = B_0$

But for a short magnet

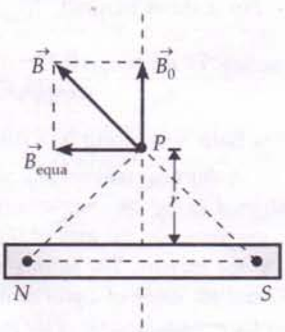
$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} \quad \therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = B_0$$

or
$$r^3 = \frac{\mu_0}{4\pi} \cdot \frac{m}{B_0} = 10^{-7} \cdot \frac{5.25 \times 10^{-2}}{0.42 \times 10^{-4}}$$
$$= 125 \times 10^{-6}$$

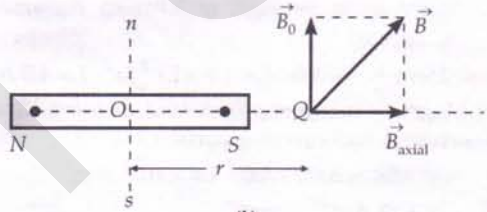
Hence $\quad r = 5 \times 10^{-2}\ \text{m} = \mathbf{5\ cm}.$

(*ii*) Figure 5.95(*b*) shows a point Q on the axis of a magnet where the resultant field is inclined at 45° with the earth's field B_0. In this condition,

$$\frac{B_{axial}}{B_0} = \tan 45° \quad \text{or} \quad B_{axial} = B_0$$



(*a*)



(*b*)

Fig. 5.95

But for a short magnet

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

$$\therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = B_0$$

or
$$r^3 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{B_0}$$
$$= 125 \times 10^{-6} \times 2 = 250 \times 10^{-6}$$

Hence $\quad r = (250)^{1/3} \times 10^{-2}\ \text{m}$
$$= 6.3 \times 10^{-2}\ \text{m} = \mathbf{6.3\ cm}.$$

5.16. *Answer the following questions :*

(*a*) *Why does a paramagnetic sample display greater magnetisation (for the same magnetising field) when cooled ?* [CBSE OD 91 ; Himachal 98C]

(*b*) *Why is diamagnetism, in contrast, almost independent of temperature ?* [Himachal 96]

(*c*) *If a toroid uses bismuth for its core, will the field in the core be slightly greater or slightly less than when the core is empty ?*

(*d*) *Is the permeability of a ferromagnetic material independent of the magnetic field ? If not, is it more for lower or higher fields ?*

(e) Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point. Why ?

(f) Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetisation of a ferromagnet ? [NCERT]

Ans. (a) The tendency to disrupt the alignment of dipoles (with the magnetising field) arising from random thermal motion is reduced at lower temperatures.

(b) The induced dipole moment in a diamagnetic sample is always opposite to the magnetising field, no matter what the internal motion of the atoms is.

(c) As bismuth is diamagnetic, so the field in the toroid with bismuth core will be slightly less than when the core is empty.

(d) No, the permeability of a ferromagnetic material is not independent of the magnetic field. This is evident from the B-H curve which has greater slope (hence greater μ) at lower fields.

(e) The proof of this important fact is based on boundary conditions of magnetic fields ($\vec{B}$ and $\vec{H}$) at the interface of two media. When one of the media has $\mu >> 1$, the field lines meet this medium nearly normally.

(f) Yes. Apart from minor differences in the strength of the individual atomic dipoles of two different materials, a paramagnetic sample with saturated magnetisation will have the same order of magnetisation. But saturation requires impractically high magnetising fields.

5.17. *Answer the following questions :*

(a) Explain qualitatively on the basis of domain picture the irreversibility in the magnetisation curve of a ferromagnet.

(b) The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetization, which piece will dissipate greater heat energy ?

(c) A system displaying a hysteresis loop such as a ferromagnet is a device for storing memory ? Explain the meaning of this statement.

(d) What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player, or for building 'memory stores' in a modern computer ?

(e) A certain region of space is to be shielded from magnetic fields. Suggest a method.

Ans. (a) In a ferromagnetic substance, the atomic dipoles are grouped together in domains. All the dipoles of a domain are aligned in the same direction and have net magnetic moment. In an unmagnetised substance these domains are randomly distributed so that the resultant magnetisation is zero. When the substance is placed in an external magnetic field, these domains align themselves in the direction of the field. Some energy is spent in the process of alignment. When the external field is removed, these domains do not come back into their original random positions completely. The substance retains some magnetisation. The energy spent in the process of magnetisation is not fully recovered. The balance of energy is lost as heat. This is the basic cause for irreversibility of the magnetisation curve of a ferromagnetic substance.

(b) Carbon steel piece, because heat lost per cycle is proportional to the area of the hysteresis loop.

(c) Magnetisation of a ferromagnet is not a single-valued function of the magnetising field. Its value for a particular field depends both on the field and also on the history of magnetisation *i.e.* how many cycles of magnetisation it has gone through etc. In other words, the value of magnetisation is a record or 'memory' of its cycles of magnetisation. If information bits can be made to correspond to these cycles, the system displaying such a hysteresis loop can act as a device for storing information.

(d) Ceramics (specially treated barium iron oxides) also called farrites.

(e) Surround the region by soft iron rings. Magnetic field lines will be drawn into the rings, and the enclosed space will be free of magnetic field. But this shielding is only approximate, unlike the perfect electric shielding of a cavity in a conductor placed in an external electric field.

5.18. *A long straight horizontal cable carries a current of 2.5 A in the direction 10° south of west to 10° north of east. The magnetic meridian of the place happens to be 10° west of the geographic meridian. The earth's magnetic field at the location is 0.33 G, and the angle of dip is zero. Locate the line of neutral points.*

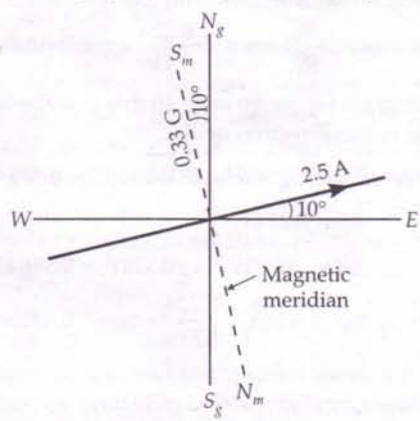


Fig. 5.96

Ans. Suppose the neutral point lies at a distance r from the cable. Then at the neutral point,

$$\frac{\mu_0 I}{2\pi r} = B_H$$

or $$r = \frac{\mu_0 I}{2\pi B_H} = \frac{4\pi \times 10^{-7} \times 2.5}{2\pi \times 0.33 \times 10^{-4}} = \mathbf{1.5 \times 10^{-2}\ m = 1.5\ cm}$$

As the direction of the magnetic field of the cable is opposite to that of $\vec{B_H}$ at points above the cable, so the line of neutral points lies parallel to and above the cable at a distance of 1.5 cm from it.

5.19. *A telephone cable at a place has four long straight horizontal wires carrying a current of 1.0 A in the same direction east to west. The earth's magnetic field at the place is 0.39 G, and the angle of dip is 35°. The magnetic declination is nearly zero. What are the resultant magnetic fields at points 4.0 cm below, and above the cable ?*

Ans. Earth's field $B = 0.39\,\text{G}, \quad \delta = 35°$

$$B_H = B\cos\delta = 0.39\cos 35° = 0.319\,\text{G}$$

$$B_V = B\sin\delta = 0.39\sin 35° = 0.224\,\text{G}$$

For the cable, we have

$I = 1.0\,\text{A}, \quad N = 4, \quad r = 4.0\,\text{cm} = 4\times10^{-2}\,\text{m}$

$\therefore$ Magnetic field produced by the cable wires is

$$B' = \frac{\mu_0 NI}{2\pi r} = \frac{4\pi\times10^{-7}\times4\times1.0}{2\pi\times4\times10^{-2}}$$

$$= 0.2\times10^{-4}\,\text{T} = 0.2\,\text{G}$$

Resultant field below the cable. At points below the cable the field B' is in the opposite direction of B_H. So the horizontal component of the resultant field is

$$R_H = B_H - B' = 0.319 - 0.2 = 0.119\,\text{G}$$

The vertical component of the earth's field remains unaffected.

$$R_V = B_V = 0.224\,\text{G}$$

$\therefore$ The magnitude of the resultant field is

$$R = \sqrt{R_H^2 + R_V^2} = \sqrt{(0.119)^2 + (0.224)^2} = \mathbf{0.254\ G}$$

The angle that R makes with the horizontal is

$$\delta = \tan^{-1}\frac{R_V}{R_H} = \tan^{-1}\frac{0.224}{0.119} = \tan^{-1} 18.8 \simeq \mathbf{62°}$$

Resultant field above the cable. In this case the field of the cable acts in the direction of B_H.

$$\therefore \quad R_H = B_H + B' = 0.319 + 0.2 = 0.519\,\text{G}$$

$$R_V = 0.224\,\text{G}$$

$$R = \sqrt{(0.519)^2 + (0.224)^2} = \mathbf{0.566\ G}$$

Angle of dip, $\delta = \tan^{-1}\dfrac{0.224}{0.519} = \tan^{-1} 0.4316 \simeq \mathbf{23°}$.

5.20. *A compass needle free to turn in a horizontal plane is placed at the centre of circular coil of 30 turns and radius 12 cm. The coil is in a vertical plane making an angle of 45° with the magnetic meridian. When the current in the coil is 0.35 A, the needle points west to east.*

(*a*) *Determine the horizontal component of the earth's magnetic field at the location.*

(*b*) *The current in the coil is reversed, and the coil is rotated about its vertical axis by an angle of 90° in the anticlockwise sense looking from above. Predict the direction of needle. Take the magnetic declination at the places to be zero.*

Ans. (*a*) Magnetic field set up at the centre of the coil is

$$B = \frac{\mu_0 NI}{2r}$$

This field acts along the axis perpendicular to the plane of the coil. As the coil is in a vertical plane making an angle of 45° with the magnetic meridian and the needle points in the *west-east* direction, it is obvious from Fig. 5.97, that the needle is oriented at an angle of 45° with the field B. Using law of sines for a triangle, we get

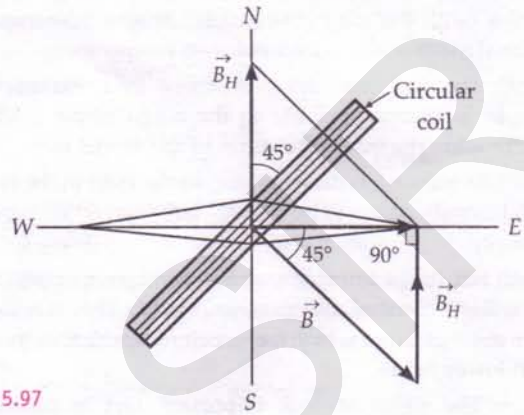


Fig. 5.97

$$\frac{B_H}{\sin 45°} = \frac{B}{\sin 90°}$$

or $$B_H = B\sin 45° = \frac{\mu_0 NI}{2r}.\sin 45°$$

$$= \frac{4\pi\times10^{-7}\times30\times0.35}{12\times10^{-2}}\times0.7071\,\text{T}$$

$$= 3.8876\times10^{-5}\,\text{T} \simeq \mathbf{0.39\ G.}$$

(*b*) The needle will reverse its original direction *i.e.*, it will point east to west.

5.21. *A magnetic dipole is under the influence of two magnetic fields. The angle between the field direction is 60° and one of the fields has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field ?*

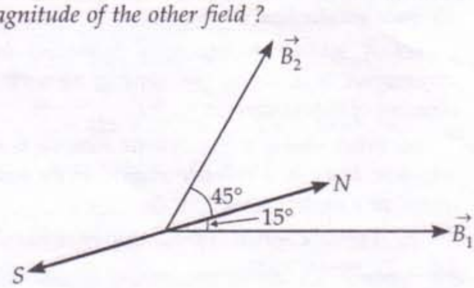


Fig. 5.98

Ans. Here $B_1 = 1.2\times10^{-2}\,\text{T}, \theta_1 = 15°$

$$\theta_2 = 60° - 15° = 45°$$

In equilibrium, $\tau_1 = \tau_2$ or $mB_1\sin\theta_1 = mB_2\sin\theta_2$

or $$B_2 = \frac{B_1\sin\theta_1}{\sin\theta_2}$$

$$= \frac{1.2\times10^{-2}\sin 15°}{\sin 45°} = \frac{1.2\times10^{-2}\times0.2588}{0.7071}$$

$$= \mathbf{4.4\times10^{-3}\,T.}$$

5.22. *A monoenergetic (18 keV) electron beam initially in the horizontal direction is subject to a horizontal magnetic field of 0.40 G normal to the initial direction. Estimate the up or down deflection of the beam over a distance of 30 cm. ($m_e = 9.11\times10^{-31}$ kg, $e = 1.60\times10^{-19}$ C).*

Ans. Kinetic energy of an electron

$$= \frac{1}{2}mv^2 = eV \qquad \therefore \qquad v = \sqrt{\frac{2eV}{m}}$$

The magnetic field provides the centripetal force to make the electron move in a circular path of radius R. Therefore,

$$\frac{mv^2}{R} = evB \quad \text{or} \quad R = \frac{mv}{eB} = \frac{m}{eB}\sqrt{\frac{2eV}{m}} = \frac{\sqrt{2meV}}{eB}$$

Here $m = 9.11\times10^{-31}$ kg, $e = 1.6\times10^{-19}$C
$V = 18$ kV $= 18\times10^3$ V, $B = 0.40$ G $= 0.40\times10^{-4}$ T

$$\therefore \quad R = \frac{\sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-19}\times18\times10^3}}{1.6\times10^{-19}\times0.40\times10^{-4}}\text{ m}$$

$$= 11.3 \text{ m.}$$

It is obvious from Fig. 5.99 that when the beam covers a distance x, its up or down deflection is y.

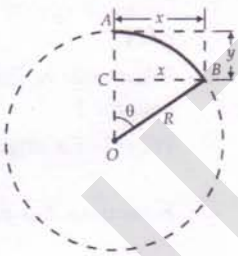


Fig. 5.99

Now from right ΔOCB

$$\sin\theta = \frac{x}{R} = \frac{0.3}{11.3} = \frac{3}{113}$$

$[\because x = 30 \text{ cm} = 0.30 \text{ m}]$

Hence up or down deflection of the electron beam is

$$y = OA - OC = R - R\cos\theta$$

$$= R[1-\cos\theta] = R[1-(1-\sin^2\theta)^{1/2}]$$

$$\simeq R\left[1-\left(1-\frac{1}{2}\sin^2\theta\right)\right]$$

$$= \frac{1}{2}R\sin^2\theta = \frac{1}{2}\times11.3\times\left(\frac{3}{113}\right)^3 \text{ m}$$

$$= 3.98\times10^{-3}\text{ m} \simeq \mathbf{4\ mm}.$$

5.23. *A sample of paramagnetic salt contains 2.0×10^{24} atomic dipoles each of dipole moment 1.5×10^{-23} JT^{-1}. The sample is placed under homogeneous magnetic field of 0.84 T and cooled to the temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K (assume Curie's law) ?*

Ans. Dipole moment of each atomic dipole,

$$m = 1.5\times10^{-23}\text{ JT}^{-1}$$

Total number of atomic dipoles, $N = 2.0\times10^{24}$

Initial total magnetic moment at temperature $T_1 = 4.2$ K is

$$M_1 = 15\% \text{ of } mN$$
$$= \frac{15}{100}\times1.5\times10^{-23}\times2.0\times10^{24}\text{ JT}^{-1} = 4.5\text{ JT}^{-1}$$

According to Curie's law,

$$M = \text{Constant}\times\frac{B}{T} \qquad \therefore \qquad \frac{M_2}{M_1} = \frac{B_2}{B_1}\times\frac{T_1}{T_2}$$

Now $B_1 = 0.84$ T, $T_1 = 4.2$ K, $B_2 = 0.98$ T, $T_2 = 2.8$ K

Hence the final dipole moment at temperature $T_2 = 2.8$ K is

$$M_2 = M_1\times\frac{B_2}{B_1}\times\frac{T_1}{T_2} = 4.5\times\frac{0.98}{0.84}\times\frac{4.2}{2.8}\text{ JT}^{-1}$$

$$= \mathbf{7.9\ JT^{-1}}.$$

5.24. *A Rowland ring of mean radius 15 cm has 3500 turns of wire wound on a ferromagnetic core of relative permeability 800. What is the magnetic field (B) in the core for a magnetising current of 1.2 A ?*

Ans. A Rowland ring is a circular ring of a magnetic material over which is wound a toroidal solenoid. The magnitude of the magnetic field in the core is given by

$$B = \mu nI$$

where $n = \dfrac{3500}{2\pi r}$, is the number of turns per unit length.

Now relative permeability,

$$\mu_r = \frac{\mu}{\mu_0}, \quad \text{so that} \quad \mu = \mu_0\mu_r$$

Hence

$$B = \mu_0\mu_r\cdot\frac{3500}{2\pi r}\cdot I$$

$$= 4\pi\times10^{-7}\times800\times\frac{3500}{2\pi\times15\times10^{-2}}\times1.2\text{ T}$$

$$= \mathbf{4.48\ T.}$$

5.25. *The magnetic moment vectors $\vec{\mu}_s$ and $\vec{\mu}_l$ associated with the intrinsic spin angular momentum $\vec{S}$ and orbital angular momentum $\vec{l}$, respectively, of an electron are predicted by quantum theory (and verified experimentally to a high accuracy) to be given by :*

$$\vec{\mu}_s = -(e/m)\vec{S}, \quad \vec{\mu}_l = -(e/2m)\vec{l}$$

Which of these relations is in accordance with the result expected 'classically' ? Outline the derivation of the classical result.

Ans. The relation $\vec{\mu}_l = -\left(\dfrac{e}{2m}\right)\vec{l}$ is in accordance with classical physics. For its derivation, refer answer to Q. 15 on page 5.13.

In contrast to μ_l / l, the magnitude of μ_s / S is e/m *i.e.*, twice the classically expected value. This latter result (verified experimentally) is an outstanding success of modern quantum theory and cannot be derived from classical physics.

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. What is a magnet ?
2. What is a natural magnet ? [Punjab 02]
3. What does the word lodestone mean ?
4. Are the two poles of a magnet equally strong ?
5. What is a magnetic dipole ? Give an example.
6. Define the term magnetic dipole moment. Give its SI unit. [CBSE OD 95 ; Haryana 02]
7. Is magnetic moment a vector ? If yes, give its direction.
8. Define a unit magnetic pole.
9. Write the SI unit of (*i*) pole strength and (*ii*) magnetic moment of a bar magnet. [CBSE F 03]
10. What is the direction of magnetic dipole moment ? [Haryana 02]
11. What is the torque experienced by a magnetic dipole moment m placed with its axis at angle θ with a uniform magnetic field B ? [ISCE 93]
12. In a uniform magnetic field, when is the torque on a magnet (*i*) maximum and (*ii*) minimum ?
13. Give the position of a magnetic dipole held in a magnetic field, where its potential energy is minimum. [CBSE OD 93]
14. In which position, the potential energy of a magnet in a uniform magnetic field is zero ?
15. On which factors does the pole strength of a magnet depend ?
16. Write the formula for the magnetic moment of a current loop. [CBSE F 02]
17. If a magnet is broken into pieces, which one of the following remains unchanged in each part – mass, moment of inertia, magnetisation ?
18. Two iron bars attract each other, no matter in which combination their ends are brought near each other. What can we say about their state of magnetisation ?
19. An electron which is a charged particle in motion has a magnetic moment. Why then does a neutron, which has no charge, have a magnetic moment ?
20. Define Bohr magneton and write its value. [Punjab 99C]
21. How much is the magnetic moment of an electron revolving in the nth orbit of hydrogen atom ?
22. Who first discovered earth's magnetism ?
23. How much is the approximate distance upto which the earth's magnetic field extends ?
24. Give an order of magnitude of earth's magnetic field near its surface.
25. What is magnetic axis of the earth ?
26. What is geomagnetic equator ?
27. What is geomagnetic meridian ? [CBSE D 92]
28. What is geographic meridian ?
29. Name the elements or parameters of earth's magnetic field. [CBSE OD 94 ; Punjab 99C ; Haryana 02]
30. Define declination at a place. [Haryana 02, 03 ; CBSE D 93C]
31. Define angle of dip (or magnetic inclination) at a place. [CBSE D 93C ; Haryana 02]
32. What will be the angle of dip at a place on the equator ? [CBSE F 95 ; D 95]
33. What is the angle of dip at magnetic poles ? [CBSE F 91]
34. How does dip angle vary from equator to poles ? [CBSE D 91 ; F 92]
35. What will be the value of the horizontal component of earth's magnetic field at earth's geomagnetic pole ? [CBSE F 95]
36. Horizontal components of Earth's magnetic field at a place is $\sqrt{3}$ times the vertical component. What is the value of angle of dip at this place ? [CBSE D 97]
37. The vertical component of Earth's mangetic field at a place is $\sqrt{3}$ times the horizontal component. What is the value of angle of dip at this place ? [CBSE D 06]
38. The needle of the dip circle is vertical at the magnetic poles. The dip circle is rotated about a vertical axis through 90°. What will be the position of the needle on the vertical circular scale ? [CBSE OD 96C]
39. Where on the surface of earth is the angle of dip (*i*) 0° and (*ii*) 90° ? [CBSE OD 11]
40. A magnetic needle, free to rotate in a vertical plane, orients itself with its axis vertical at a certain place on the earth. What are the values of :
    (*a*) horizontal component of earth's magnetic field ?
    (*b*) angle of dip, at this place ? [CBSE F 12]

41. The horizontal component of the earth's magnetic field at a place is B and angle of dip is 60°. What is the value of vertical component of earth's magnetic field at equator ? [CBSE D 12]

42. Where on the earth's surface is the vertical component of the earth's magnetic field zero ? [CBSE OD 11 ; D 13C]

43. What are isogonic lines ?

44. What are isoclinical lines ?

45. What is an aclinic line ?

46. What are isodynamic lines ?

47. Define neutral point in the magnetic field of a bar magnet. [Punjab 02]

48. A magnet is placed with the north pole towards the north of the earth. Predict the position of the neutral points.

49. What is the difference between a compass needle and dip needle ?

50. At a certain location in Africa, a compass needle points 15° West of the geographic north. What is the angle of declination at that point ? [Haryana 97]

51. Torques τ_1 and τ_2 are required for a magnetic needle to remain perpendicular to the magnetic fields B_1 and B_2 at two different places. What is the ratio of the magnetic fields at the two places ?

52. Give the order of magnetic moment of an atom.

53. What is induced magnetism ?

54. What is a magnetising field ?

55. Define magnetic intensity. Give its SI unit.

56. Define magnetic induction. Give its SI unit.

57. Define magnetisation of a material. Give its SI unit.

58. Define magnetic permeability. State its SI unit. [ISCE 98]

59. What is relative permeability ?

60. Define magnetic susceptibility. [Haryana 01]

61. Relative permeability of a material, $\mu_r = 0.5$. Identify the nature of the magnetic material and write its relation to magnetic susceptibility. [CBSE D 14C]

62. What is the one quantum of atomic dipole moment called ?

63. What is a diamagnetic substance ? [Punjab 97]

64. Which of the following substances are diamagnetic ?
Bi, Al, Na, Cu, Ca and Ni [CBSE D 13]

65. What is a paramagnetic substance ? [Punjab 98C, 99]

66. Which of the following substances are paramagnetic ? Bi, Al, Cu, Ca, Pb, Ni [CBSE D 13]

67. What is a ferromagnetic substance ?

68. Relative permeability (μ_r) of a material has a value lying $1 < \mu_r < 1 + \varepsilon$ (where ε is a small quantity). Identify the nature of the magnetic materials. [CBSE D 14C]

69. Relative permeability of a material $\mu_r = 400$. Identify the nature of the magnetic material. [CBSE D 14C]

70. The permeability of bismuth is 0.9983. To which class of magnetic materials, does bismuth belong ? [CBSE D 11]

71. How does the intensity of magnetisation of a paramagnetic sample vary with temperature ? [CBSE OD 2000]

72. Will the neon gas be diamagnetic or paramagnetic ? Give reason.

73. What is curie point ?

74. Give two essential characteristics of a material used for preparing an electromagnet. [CBSE F 04]

75. Define hysteresis.

76. What does the area of hysteresis loop indicate ?

77. The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy ?

78. What is the basic use of a hysteresis curve ?

79. Name two alloys commonly used for making cores of transformers.

80. Name two magnetic materials commonly used for making permanent magnets.

81. Suggest two methods to destroy the magnetism of a magnet. [Punjab 02]

82. What are permanent magnets ? Give one example. [CBSE D 13]

83. How does the magnetic permeability μ_r differ for dia, para and ferromagnetic materials ? [ISCE 03]

84. Mention the two characteristic properties of the material suitable for making core of a transformer. [CBSE OD 12]

85. A (hypothetical) bar magnet (AB) is cut into two equal parts. One part is now kept over the other, so that pole C_2 is above C_1. If M is the magnetic moment of the original magnet, what would be the magnetic moment of the combination so formed ? [CBSE Sample Paper 08]

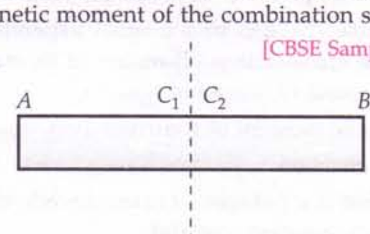


Fig. 5.100

86. An electric current of 0.25 A flows in a loop of radius 0.2 cm. Calculate the magnitude of the magnetic dipole moment of the dipole formed. [ISCE 02]

87. Two circular loops, of radii r and $2r$, have currents, I and $I/2$ flowing through them in clockwise and anticlockwise sense respectively. If their equivalent magnetic moments are M_1 and M_2 respectively, state the relation between M_1 and M_2. [CBSE Sample Paper 2011]

Answers

1. A magnet is a piece of material that has both attractive and directive properties. It attracts small pieces of iron, nickel, cobalt etc.
2. Natural magnet is an iron ore called magnetite. Chemically, it is a black iron oxide of formula Fe_3O_4.
3. The word lodestone means a leading stone. It represents the directive behaviour of a magnet.
4. Yes. The two poles of a magnet are always equally strong.
5. Refer to point 12 of Glimpses on page 5.78.
6. Refer to point 13 of Glimpses on page 5.78.
7. Yes, magnetic moment is a vector. Its direction is from south pole to north pole of the magnet.
8. A magnetic pole which when placed in vacuum at a distance of 1 m from an identical pole repels it with a force of 10^{-7} newton is called unit magnetic pole.
9. (*i*) The SI unit of pole strength is ampere metre (Am).
   (*ii*) The SI unit of magnetic moment is ampere metre^2 (Am^2) or joule/tesla (JT^{-1}).
10. The direction of magnetic dipole moment is from S-pole to N-pole of the magnet.
11. Torque, $\tau = mB \sin \theta$.
12. (*i*) Torque is maximum when the magnet lies perpendicular to the direction of the magnetic field.
    (*ii*) Torque is minimum (zero) when the magnet lies along the direction of the magnetic field.
13. The P.E. of a magnetic dipole is minimum when its dipole moment $\vec{m}$ is parallel to the magnetic field $\vec{B}$.
14. The P.E. of a magnetic dipole is zero when its dipole moment $\vec{m}$ is perpendicular to the field $\vec{B}$.
15. The pole strength of a magnet depends on (*i*) its area of cross-section (*ii*) nature of its material and (*iii*) its state of magnetisation.
16. Magnetic moment of a current loop, $m = IA$.
17. Magnetisation.
18. One rod is a permanent magnet while the other is an unmagnetised iron rod.
19. Like mass and charge, magnetic moment is a basic property, one may consider neutron as a particle containing equal amounts of positive and negative charges spinning in opposite directions.
20. Refer to point 22 of Glimpses on page 5.79.
21. $\mu_l = n\mu_B = n\left(\frac{eh}{4\pi m_e}\right)$.
22. William Gilbert.
23. Earth's magnetic field extends upto a distance of 32000 km, which is about five times the radius of the earth.
24. It is 10^{-4} T or 1 gauss (G).
25. The straight line passing through the magnetic north and south poles of the earth is called magnetic axis of the earth.
26. It is the great circle on the earth perpendicular to the magnetic axis.
27. Refer to point 26 of Glimpses on page 5.79.
28. Refer to point 27 of Glimpses on page 5.79.
29. The elements of earth's magnetic field are
    (*i*) Declination (*ii*) Dip
    (*iii*) Horizontal component of earth's magnetic field.
30. Refer to point 29 of Glimpses on page 5.79.
31. Refer to point 30 of Glimpses on page 5.79.
32. Angle of dip at the equator, $\delta = 0°$.
33. Angle of dip at magnetic poles, $\delta = 90°$.
34. The dip angle increases from 0° to 90° as one moves from magnetic equator to poles.
35. At poles, $\delta = 90°$
    ∴ Horizontal component,
    $$B_H = B\cos\delta = B\cos 90° = 0.$$
36. Here $\tan\delta = \frac{B_V}{B_H} = \frac{B_V}{\sqrt{3}\,B_V} = \frac{1}{\sqrt{3}}$
    ∴ Angle of dip, $\delta = \mathbf{30°}$.
37. Here $\tan\delta = \frac{B_V}{B_H} = \frac{\sqrt{3}B_H}{B_H} = \sqrt{3}$
    ∴ $\delta = \mathbf{60°}$.

38. 90°.

39. (*i*) Angle of dip is 0° at magnetic equator. (*ii*) Angle of dip is 90° at magnetic poles of the earth.

40. (*a*) Horizontal component of earth's magnetic field is zero.
 (*b*) Angle of dip at the given place is 90°.

41. At the magnetic equator, $B_V = 0$.

42. At the magnetic equator, $B_V = 0$.

43. The lines joining the places of equal declination are called isogonic lines.

44. The lines joining the places of equal dip or inclination are called isoclinical lines.

45. The line of zero dip is called aclinic line or magnetic equator.

46. The line joining the places having the same value of the horizontal component of earth's magnetic field. are called isodynamic lines.

47. Neutral point is that point where the magnetic field due to a magnet is equal and opposite to the horizontal component of the earth's magnetic field. The resultant magnetic field at the neutral point is zero.

48. The neutral points lie on equatorial line on either side of the magnet.

49. A compass needle is free to rotate about a vertical axis in a horizontal plane while a dip needle is free to rotate about a horizontal axis in a vertical plane.

50. $\alpha = 15°$, west of the geographic north.

51. Torque on a magnetic needle,

$$\tau = mB \sin 90° = mB \quad i.e., \quad \tau \propto B$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{\tau_1}{\tau_2}.$$

52. 10^{-23} Am^2.

53. When a piece of magnetic material is placed in a magnetic field, the material gets magnetised. The magnetism so produced in the material is called induced magnetism.

54. Refer to point 34 of Glimpses on page 5.80.

55. Refer to point 35 of Glimpses on page 5.80.

56. Refer to point 37 of Glimpses on page 5.80.

57. Refer to point 36 of Glimpses on page 5.80.

58. Refer to point 38 of Glimpses on page 5.80.

59. Refer to point 39 of Glimpses on page 5.80.

60. Refer to point 40 of Glimpses on page 5.80.

61. As $0 < \mu_r < 1$, the material is diamagnetic.

$$\mu_r = 1 + \chi_m.$$

62. Bohr magneton.

63. Refer to point 42 of Glimpses on page 5.80.

64. Bi and Cu.

65. Refer to point 43 of Glimpses on page 5.80.

66. Al and Ca.

67. Refer to point 44 of Glimpses on page 5.80.

68. Paramagnetic.

69. Ferromagnetic.

70. As the permeability of bismuth is less than 1, so it is diamagnetic.

71. $M \propto \frac{1}{T}$.

72. As there are no unpaired electrons in neon atoms, so neon gas is diamagnetic.

73. Curie point is the temperature above which a ferromagnetic substance becomes paramagnetic.

74. The material used for making an electromagnet must have (*i*) high permeability and (*ii*) low retentivity.

75. The phenomenon of lagging of magnetic induction behind the magnetising field in a magnetic material is called hysteresis.

76. The area of the hysteresis loop gives a measure of the energy wasted in a sample when it is taken through a cycle of magnetisation.

77. The carbon steel piece will dissipate greater heat energy as its hysteresis loop has greater area.

78. The study of hysteresis curve of a material gives its hysteresis loss, retentivity and coercivity. This knowledge helps in selecting materials for making cores of transformers, permanent magnets and electromagnets.

79. (*i*) *Mu-metal*, an alloy of nickel, iron, copper and chromium.
 (*ii*) *Peum metal*, an alloy of iron and nickel.

80. (*i*) *Cobalt steel*, an alloy of cobalt, tungsten and carbon
 (*ii*) *Alnico*, an alloy of iron, nickel, aluminium, cobalt and copper.

81. (*i*) By heating the magnet.
 (*ii*) By applying magnetic field in the reverse direction.

82. The substances which at room temperature retain their ferromagnetism for a long time are called permanent magnets *e.g.*, cobalt, steel, alnico and ticonal.

83. For diamagnetic substances, $\mu_r < 1$

For paramagnetic substances, $\mu_r > 1$

For ferromagnetic substances, $\mu_r >> 1$

84. The core material of transformer must have (*i*) high initial permeability and (*ii*) lower hysteresis loss.

85. Magnetic moment of the combination will be zero. As poles C_1 and C_2 will be of opposite nature, the magnetic moments of the two parts get aligned in opposite direction in the combination. Hence $(M/2)-(M/2)=0$.

86. $m = IA = I \times \pi r^2 = 0.25 \times 3.14 \times (0.2 \times 10^{-2})^2$
$= 3.14 \times 10^{-6}\ \text{Am}^2$

87. $\vec{M_1} = -\frac{1}{2}\vec{M_2}$

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. Define the terms magnet and magnetism. What is the origin of the word magnetism ?
2. What is meant by the terms magnetic field and magnetic field intensity ? [Haryana 94]
3. State four basic properties of magnets. [Punjab 02]
4. Define the terms magnetic poles, magnetic axis, magnetic equator and magnetic length with reference to a bar magnet.
5. State Coulomb's law of magnetic forces. Hence define a unit magnetic pole.
6. What is a magnetic dipole ? Define magnetic dipole moment. Give its SI unit.
7. What is a uniform magnetic field ? Give an example and sketch lines of force of such a field.
8. What are magnetic lines of force ? Sketch the magnetic field lines of a bar magnet. [CBSE OD 90C]
9. What are magnetic lines of force ? Give their important properties. [Haryana 94]
10. What is the name given to the curves the tangent to which at any point gives the direction of the magnetic field at that point ? Can two such curves intersect each other ? Justify your answer. [CBSE Sample Paper 97]
11. Derive an expression for the torque experienced by a magnetic dipole in a uniform magnetic field. Hence obtain the expression for the potential energy of the dipole. [CBSE F 03]
12. Derive an expression for the torque on a magnetic dipole placed in a magnetic field and hence define magnetic dipole moment. [Haryana 96 ; Himachal 2000]
13. Derive an expression for the potential energy of a bar magnet when placed in a uniform magnetic field. [Himachal 01]
14. Derive an expression for the intensity of the magnetic field at an axial point of a short magnetic dipole.
15. Derive an expression for the magnetic field due to a magnetic dipole in broad-side on position at a distance r from its centre. The length of the dipole is $2l$ and its magnetic moment is m. [ISCE 98]
16. A current loop behaves as a magnetic dipole. Obtain an expression for the magnetic dipole moment of a circular current loop. State the rule used to find the direction of the magnetic dipole moment. [CBSE F 95]
17. Show that the electron revolving around the nucleus in an orbit of radius r with speed v has magnetic moment $evr/2$. Hence using Bohr's postulate of angular momentum, obtain the expression for the magnetic moment of hydrogen atom in its ground state. [CBSE OD 13C]
18. Give an experimental evidence in support of the existence of earth's magnetic field.
19. Compare the magnetic fields of a bar magnet and current carrying solenoid. [Punjab 01]
20. Explain with the help of diagram (*i*) magnetic declination and (*ii*) magnetic of dip at a place. In what direction will a compass needle point when kept at the (*i*) poles and (*ii*) equator ? [CBSE D 04, 05, 15C]
21. Name three elements required to specify the earth's magnetic field at a given place. Draw a labelled diagram to define these elements. Explain briefly how these elements are determined to find out the magnetic field at a given place on the surface of Earth. [CBSE D 04 C, 08C]
22. Name and define the magnetic elements of earth's magnetic field at a place. Derive an expression for the angle of dip in terms of the horizontal component and the resultant magnetic field of the earth at a given place. [CBSE D 01, 02]
23. Define neutral point. Locate the positions of neutral points, when a small bar magnet is placed with its north pole (*i*) towards the north of the earth and (*ii*) towards the south of the earth. [CBSE D 95C]
24. Sketch the magnetic field pattern of a bar magnet placed in the magnetic meridian with its N-pole pointing north. Show the neutral points. How will you determine the dipole moment of the magnet ? [Punjab 2000]
25. Define neutral point. Sketch the lines of magnetic field when a small bar magnet is placed with N-pole towards the south of the earth. Indicate the position of neutral points. [CBSE OD 02C]
26. What is "Aurora Borealis" (or northern lights) ? Can it be seen in India ?

27. What is relative permeability of a magnetic material ? How is it related to the magnetic susceptibility ? [ISCE 97]
28. State and explain Curie law of magnetism. [CBSE OD 97 ; Punjab 2000, 01]
29. What are dia, para and ferromagnetic substances ? Give one example of each. [Haryana 02]
30. Classify materials on the basis of their behaviour in a magnetic field. Under which category does iron come ? How does the magnetic property of iron change with increase of temperature ? [CBSE D 95]
31. Distinguish between diamagnetic and ferromagnetic substances in respect of (*i*) intensity of magnetisation, (*ii*) behaviour in a non-uniform magnetic field and (*iii*) susceptibility. [ISCE 95 ; CBSE D 03]
32. What are diamagnetic substances ? Explain the origin of diamagnetism on the basis of electron theory. [Haryana 01]
33. What are paramagnetic substances ? Explain the origin of paramagnetism.
34. What are ferromagnetic substances ? Explain briefly domain theory to explain ferromagnetism. [Haryana 02]
35. Explain the phenomenon of hysteresis in magnetic materials. Draw a hysteresis loop showing remanence and coercive force. [ISCE 96 ; Haryana 01]
36. Define the terms retentivity and coercivity. [Himachal 97]
37. Write two characteristic properties to distinguish between diamagnetic and paramagnetic materials. [CBSE OD 05]
38. Draw a hysteresis loop with axes labelled and showing remanence and coercive force. What does the area of the loop represent ? [ISCE 02]
39. Define the term magnetic dipole moment of a current loop. Write the expression for the magnetic moment when an electron revolves at a speed 'v', around an orbit of radius 'r' in hydrogen atom. [CBSE OD 08, 10]
40. Permanent magnets are made of special alloys while the cores of temporary magnets are made of soft iron. Why ? [ISCE 2000]
41. Depict the field-line pattern due to a current carrying solenoid of finite length.
    (*i*) In what way do these lines differ from those due to an electric dipole ?
    (*ii*) Why can't two magnetic field lines intersect each other ? [CBSE F 09]
42. Derive an expression for the magnetic moment ($\vec{\mu}$) of an electron revolving around the nucleus in terms of its angular momentum ($\vec{l}$). What is the direction of the magnetic moment of the electron with respect to its angular momentum ? [CBSE OD 14C]
43. Out of the following, indentify the materials which can be classified as : (*i*) paramagnetic, (*ii*) diamagnetic :
    (*a*) Aluminium (*b*) Bismuth (*c*) Copper (*d*) Sodium
    Write one property to distinguish between paramagnetic and diamagnetic materials. [CBSE D 09C]
44. (*i*) How does angle of dip change as one goes from magnetic pole to magnetic equator of the earth ?
    (*ii*) A uniform magnetic field gets modified as shown below when two specimens X and Y are placed in it. Identify whether specimens X and Y are diamagnetic, paramagnetic or ferromagnetic.

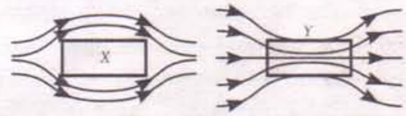


Fig. 5.101

    (*iii*) How is the magnetic permeability of specimen X different from that of specimen Y ? [CBSE F 09 ; D 09C]
45. (*a*) How is an electromagnet different from a permanent magnet ? (*b*) Write two properties of a material which make it suitable for making electromagnets. [CBSE OD 14C]

Answers

1. Refer to point 1 of Glimpses.
2. **Magnetic field.** The space around a magnet within which its influence can be experienced is called its magnetic field.

   Magnetic field intensity. The magnetic field intensity at any point in a magnetic field is defined as the force experienced by a unit north pole placed at that point. It is a vector quantity whose direction is same as the direction in which unit north pole would tend to move if free to do so.
3. Refer to point 4 of Glimpses.
4. Refer answer to Q. 4 on page 5.2.
5. Refer answer to Q. 5 on page 5.3.
6. Refer answers to Q. 6 on page 5.3 and Q. 7 on page 5.4.

7. Refer answer to Q.4 on page 5.3. As shown in Fig. 5.102, the lines of force of a uniform magnetic field are parallel to one another.

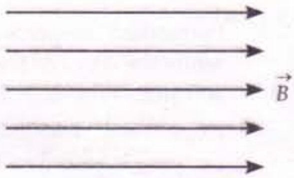


Fig. 5.102

8. Refer answer to Q.8 on page 5.6. For the lines of force of a bar magnet, see Fig 5.24 on page 5.18.
9. Refer answer to Q. 8 on page 5.6.
10. Such curves are called magnetic lines of force. No two such lines of force can intersect. If they do so, then there will be two tangents and hence two directions of the magnetic field at the point of intersection which is impossible.
11. Refer answers to Q. 12 and Q. 13 on page 5.11.
12. Refer answer to Q. 12 on page 5.11.
13. Refer answer to Q. 13 on page 5.11.
14. Refer answer to Q. 10 on page 5.7.
15. Refer answer to Q. 11 on page 5.7.
16. Refer answer to Q. 14 on page 5.12.
17. Refer answer to Q. 15 on page 5.13.
18. Refer answer to Q. 20 on page 5.20.
19. Figure 5.24 shows the lines of force of a bar magnet while Fig. 5.25 shows the lines of force of a straight solenoid. The two patterns have a striking resemblance. So a solenoid behaves like a bar magnet. The two ends of both have north and south polarity.
20. Refer answer to Q. 23 on page 5.21. At the poles, the compass needle will stay in any direction. At the equator, it will come to rest in the magnetic north-south direction.
21. Refer answer to Q. 23 on page 5.21.
22. Refer answer to Q. 23 on page 5.21.
23. Refer answer to Q. 28 on page 5.23.
24. Refer answer to Q. 28 on page 5.23.
25. See Fig. 5.35 on page 5.24.
26. Refer answer to Q. 26 on page 5.23.

    This phenomenon cannot be seen anywhere in India as it occurs only near the magnetic poles.
27. Refer to points 39 and 40 of Glimpses.
28. Refer answer to Q. 32 on page 5.31.
29. Refer answer to Q. 30 on page 5.30.
30. Refer answer to Q. 30 on page 5.30.

    Iron is a ferromagnetic substance. As temperature increases, its magnetisation decreases due to randomisation of its domains.
31. See Table 5.3 on page 5.35.
32. Refer answer to Q. 31 on page 5.30.
33. Refer answer to Q. 32 on page 5.31.
34. Refer answer to Q. 33 on page 5.31.
35. Refer answer to Q. 38 on page 5.36.
36. Refer answer to Q. 38 on page 5.36.
37. (*i*) In a uniform magnetic field, a rod of diamagnetic material aligns itself perpendicular to the field while that of paramagnetic material aligns itself parallel to the field.

    (*ii*) The magnetic susceptibility of a diamagnetic material is small and negative while that of paramagnetic material, the value is small and positive.
38. Refer answer to Q. 38 on page 5.36.
39. Refer answer to Q. 14 on page 5.12 and Q. 15 on page 5.13.
40. Permanent magnets are made of special alloys like steel because of their high coercivity. The cores of temporary magnets are made of soft iron because of its high permeability and low retentivity.
41. See Fig. 5.25 on page 5.18.

    (*i*) The magnetic lines of force of a solenoid form closed loops while the electric lines of force of an electric dipole start from the positive charge and end at the negative charge.

    (*ii*) Refer answer to Q.10 on page 5.74.
42. Refer answer to Q.15 on page 5.13.
43. (*i*) *Paramagnetic* : Aluminium and sodium.

    (*ii*) *Diagmagnetic* : Bismuta and copper.

    Refer answer to Q.37 above.
44. (*i*) The angle of dip decereases from 90° to 0° as one goes from magnetic pole to magnetic equator of the earth.

    (*ii*) X is diamagnetic as it expels field lines.

    Y is paramagnetic or ferromagnetic as it pulls in field lines.

    (*iii*) Magnetic permeability of X is less than 1 and of Y is greater than 1.
45. (*a*) An electromagnet consists of a core made of a ferromagnetic material placed inside a solenoid. It behaves like a strong magnet when current flows through the solenoid and effectively loses its magnetism when the current is switched off.

    The substances which at room temperature retain their ferromagnetism, after being magnetised once, are called permanent magnets.

    (*b*) The core of an electromagnet must have (*i*) high permeability (*ii*) low retentivity and (*iii*) low coercivity.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. (*i*) What is the relationship between the current and the magnetic moment of a current carrying circular loop ? Use the expression to derive the relation between the magnetic moment of an electron moving in a circle and its related angular momentum ?

   (*ii*) A muon is a particle that has the same charge as an electron but is 200 times heavier than it. It we had an atom in which the muon revolves around a proton instead of an electron, what would be the magnetic moment of the muon in the ground state of such an atom ? [CBSE Sample Paper 08]

2. Explain the possible causes of earth's magnetic field. [Punjab 98C, 01]

3. Distinguish the magnetic properties of a dia-, para- and ferromagnetic substances in terms of (*i*) susceptibility, (*ii*) magnetic permeability and (*iii*) coercivity. Give one example of each of these materials.

   Draw the field lines due to an external magnetic field near a (*i*) diamagnetic, (*ii*) paramagnetic substance. [CBSE OD 07]

4. (*a*) How does a paramagnetic material behave in the presence of an external magnetic field ? Explain with the help of an appropriate diagram.

   (*b*) What happens when the temperature of a paramagnetic sample is lowered ?

   (*c*) To which of the two – a polar dielectric or a non-polar dielectric – does a paramagnetic material correspond ? Justify your answer. [CBSE Sample Paper 2011]

5. (*a*) A small compass needle of magnetic moment 'm' is free to turn about an axis perpendicular to the direction of uniform magnetic field 'B'. The moment of inertia of the needle about the axis is 'I'. The needle is slightly disturbed from its stable position and then released. Prove that it executes simple harmonic motion. Hence deduce the expression for its time period.

   (*b*) A compass needle, free to turn in a vertical plane orients itself with its axis vertical at a certain place on the earth. Find out the values of (*i*) horizontal components of earth's magnetic field and (*ii*) angle of dip at the place. [CBSE D 13]

Answers

1. (*i*) Magnetic moment of a circular current loop

   = Current × Area of the loop or $m = IA$

   For magnetic moment of an electron, refer answer to Q. 15 on page 5.13.

   (*ii*) Magnetic moment of an electron in ground state,

   $$\mu_l\,(\text{electron}) = \frac{eh}{4\pi m_e}$$

   As mass of a muon is 200 times the mass of an electron, *i.e.,* its magnetic moment in the ground state is

   $$\mu_l\,(\text{muon}) = \frac{eh}{4\pi \times 200\, m_e} = \frac{1.6 \times 10^{-19} \times 6.6 \times 10^{-34}}{4\pi \times 200 \times 9.1 \times 10^{-31}}$$

   $$= \mathbf{4.63 \times 10^{-26}\ Am^2}.$$

2. Refer answer to Q. 23 on page 5.21.

3. See Table 5.3 on page 5.35 and see Fig. 5.76 on page 5.54.

4. (*a*) Refer answer to Q. 32 on page 5.31.

   (*b*) As the temperature of a paramagnetic substance is lowered, its atomic dipoles tend to get aligned with the magnetising field. So its magnetisation increases until it reaches a saturation value at a stage when all the dipoles get perfectly aligned with the field.

   (*c*) Paramagnetic material is a kind of polar dielectric. This is because the atoms/molecules of such a material have non-zero magnetic moment.

5. (*a*) Refer answer to Q. 46 on page 5.42.

   (*b*) (*i*) Horizontal component of earth's magnetic field is zero. $B_H = B\cos 90° = 0$.

   (*ii*) As the compass needle stands vertically, angle of dip (δ) = **90°**.

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. Once Indu noticed that her aunt was suffering from severe joint pain and could not take any pain killer being allergic to them. In her quest to help her aunt, Indu found the use of magnets. She read Dr. Philpott's work on magnetic therapy. She found that many people are negative magnetic field deficient due to electromagnetic pollution. When the body is supplemented with negative field energy, it brings about quick healing. Indu took her aunt to a magnetic therapy centre on regular basis. Her aunt recovered at a great pace.

   (*a*) What are the values being highlighted by Indu ?

   (*b*) What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5 cm at a distance of 50 cm from the midpoint ? The magnetic moment of the bar magnet is 0.40 Am^2.

2. Rohit's mother put many clothes for washing in the washing machine. She could not start the washing machine as an indicator was showing that the lid was not closed. On seeing his mother struggling, Rohit tried his best to close the lid but soon he realised that the closing mechanism was not a manual system but a magnetic system. He went to the shop and purchased a magnetic closer. He replaced the older closer by the new one. The machine started working. The mother was happy that her son helped her to save three hundred rupees.
   (*a*) What are the values being displayed by Rohit ?
   (*b*) What are the values being shown by Rohit's mother ?
   (*c*) A magnetised needle in a uniform magnetic field experiences a torque but no net force. An iron nail near a bar magnet, however, experiences a force of attraction in addition to a torque. Why ?
3. Shubham, once watching Discovery Channel, saw that certain organisms have the ability to sense the field lines of earth's magnetic field. By following these lines, they can travel from one place to another. Shubham, in his quest to know more about earth's magnetic field, brought a magnetic compass. By knowing the direction in which the compass comes to rest, he found the magnetic meridian. Then he fixed the compass on a cardboard and placed it vertically along the magnetic meridian. By measuring the inclination of the compass with the horizontal, he found the angle of dip at the given place.
   (*a*) What values did Shubham have ?
   (*b*) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole ?
4. While watching a science channel on TV, Renu once saw that a spectacular display of light like the one during commonwealth games could be seen in the night sky at high altitudes near the polar regions. She got surprised and consulted her Physics teacher to know more about this phenomenon. She was explained that this phenomenon called aurora is caused when the charged particles of the solar wind get attracted by the magnetic poles of the earth and there they ionise the atmospheric atoms or molecules emiting green and pink light – the glow of the aurora. The aurora in the northern hemisphere is called aurora borealis and that in the southern hemisphere is called aurora australis. Renu shared this knowledge with her classmates also.
   (*a*) What values did Renu have ?
   (*b*) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion ?
5. Two friends Deepa and Shilpa were good dancers and used to perform in the school functions by using their cassette player. One day when they were performing, the tape got stuck up and the music stopped. But Deepa was determined not to let down the performance, so she decided to sing the song instead of dancing and Shilpa completed the dance.
   (*a*) What were the values displayed by Deepa and Shilpa ?
   (*b*) What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player, or building 'memory stores' in a modern computer ?

Answers

1. (*a*) Love, sympathy, diligence, punctuality and regularity, maturity and responsibility.
   (*b*) Refer to the solution of Example 7 on page 5.8.
2. (*a*) Sympathy, responsibility, helping & self-reliance.
   (*b*) Appreciation, thankfulness & economical nature.
   (*c*) In the first case, the magnetic field is uniform, and forces acting on the two ends of the needle are equal and opposite. So the net force is zero. However, a torque acts on the needle.

   In the case of iron nail, there is an induced magnetism. The induced (say) south pole in the nail, being closer to the north pole of the bar magnet, experiences a larger attractive force than the induced north pole. So the nail experiences both a net attractive force and a torque.
3. (*a*) Nature of appreciation, curiosity, diligence, self-reliance and creative skill.
   (*b*) Earth's magnetic field is exactly vertical at the poles and so the horizontal component of earth's magnetic field is zero which makes the compass needle point in any direction at the geomagnetic north or south pole.
4. (*a*) Nature of appreciation, diligence, research mindedness and communication skills.
   (*b*) At large distances, the field gets modified due to the field of ions in motion (in the earth's ionosphere). The field of these ions, in turn, is sensitive to extra terrestrial disturbances such as solar wind.
5. (*a*) Team spirit, confidence, determination and courage.
   (*b*) Ceramics (specially treated barium iron oxides) also called ferrites.

COMPETITION SECTION

Magnetism

GLIMPSES

1. **Magnets and magnetism.** A magnet is a piece of material that has both attractive and directive properties. It attracts small pieces of iron, nickel, cobalt, etc. This property of attraction is called magnetism.

2. **Natural magnets.** A natural magnet is an iron ore called lodestone (leading stone) or magnetite. It is a black iron oxide, Fe_3O_4. Natural magnets were found as early as the sixth century B.C. in the province of Magnesia in ancient Greece, from which the word magnetism derives its name.

3. **Artificial magnets.** Pieces of iron and other magnetic materials can be made to acquire the properties of natural magnets. Such magnets are called artificial magnets.

4. **Basic properties of magnets.** These are as follows :

   (*i*) *Attractive property.* A magnet attracts small pieces of iron, nickel, cobalt, etc.

   (*ii*) *Directive property.* A freely suspended magnet aligns itself nearly in the geographic north-south direction.

   (*iii*) *Like poles repel and unlike poles attract.* This is a fundamental law of magnetic poles.

   (*iv*) *Magnetic poles exist in pairs.* Isolated magnetic poles do not exist. If we break a magnet into two pieces, we get two smaller dipole magnets.

5. **Magnetic field.** The space around a magnet within which its influence can be experienced is called its magnetic field.

6. **Uniform magnetic field.** A magnetic field in a region is said to be uniform if it has same magnitude and direction at all points of that region.

7. **Magnetic poles.** These are the regions of apparently concentrated magnetic strength in a magnet where the magnetic attraction is maximum.

8. **Magnetic axis.** The line passing through the poles of a magnet is called its magnetic axis.

9. **Magnetic equator.** The line passing through the centre of the magnet and at right angles to the magnetic axis is called the magnetic equator of the magnet.

10. **Magnetic length.** The distance between the two poles of a magnet is called its magnetic length. It is slightly less than the geometrical length of the magnet.

11. **Coulomb's law of magnetic force.** This law states that the force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them. If q_{m_1} and q_{m_2} are the pole strengths of two magnetic poles separated by distance r, then the force of attraction or repulsion between them is

$$F = \frac{\mu_0}{4\pi} \cdot \frac{q_{m_1} q_{m_2}}{r^2}$$

where μ_0 is the permeability of free space and its value is $4\pi \times 10^{-7}$ henry/metre or TmA^{-1}.

12. Magnetic dipole. Any arrangement of two equal and opposite magnetic poles separated by a small distance is called a magnetic dipole. A bar magnet and a current-carrying loop are magnetic dipoles.

13. Magnetic dipole moment. It is equal to the product of the pole strength (q_m) and the magnetic length ($2l$) of the magnet.

$$m = q_m \times 2l$$

The SI unit of magnetic dipole moment is Am^2 or JT^{-1}.

14. Magnetic lines of force. A magnetic line of force may be defined as the curve the tangent to which at any point gives the direction of the magnetic field at that point. It may also be defined as the path along which a unit north pole would tend to move if free to do so.

15. Properties of lines of force. These are as follows :

(*i*) Magnetic lines of force are closed curves which start in air from the N-pole and end at the S-pole and then return to the N-pole through the interior of the magnet.

(*ii*) The lines of force never cross each other.

(*iii*) They start from, and end on the surface of the magnet normally.

(*iv*) The lines of force have a tendency to contract lengthwise and expand sidewise. This explains attraction between unlike poles and repulsion between like poles.

(*v*) The relative closeness of the lines of force is a measure of the strength of the magnetic field which is maximum at the poles.

16. Magnetic field of a bar magnet at an axial point (end-on position).

(*i*) $B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2}$,

where r is the distance of the point from the centre of the magnet.

(*ii*) For a short magnet, $l << r$,

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

The magnetic field at any axial point of magnetic dipole is in the same direction as that of its magnetic dipole moment.

17. Magnetic field of a bar magnet at an equatorial point (Broadside-on position).

(*i*) $B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}}$

(*ii*) For short magnet, $l << r$,

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}.$$

The magnetic field at any equatorial point of a magnetic dipole is in the direction opposite to that of its magnetic dipole moment.

18. Torque on a magnet in a magnetic field. If a magnet of dipole moment $\vec{m}$ is placed in a magnetic field $\vec{B}$ making an angle θ with it, then torque acting on the magnet is

$$\tau = mB \sin\theta$$

In vector notation, $\vec{\tau} = \vec{m} \times \vec{B}$

The effect of the torque is to align the dipole parallel to the field $\vec{B}$.

If $\theta = 90°$, then $\tau = mB$

Hence the magnetic dipole may be defined as the torque acting on a magnetic dipole placed perpendicular to a uniform magnetic field of unit strength.

19. Potential energy of a magnetic dipole in a magnetic field. When a magnetic dipole is rotated in a magnetic field against the torque from initial position θ_1 to final position θ_2, the work done or the potential energy stored is given by

$$W = U = -mB(\cos\theta_2 - \cos\theta_1)$$

P.E. is zero when $\vec{m} \perp \vec{B}$. Hence P.E. of the dipole in any orientation θ is

$$U = -mB\cos\theta = -\vec{m} \cdot \vec{B}$$

Special Cases

1. When $\theta = 0°$, $U = -mB$. Thus the P.E. of a dipole is minimum when $\vec{m}$ is parallel to $\vec{B}$. This is the position of stable equilibrium.
2. When $\theta = 90°$, $U = 0$.
3. When $\theta = 180°$, $U = +mB$.

Thus the P.E. of the dipole is maximum when $\vec{m}$ is antiparallel to $\vec{B}$. This is the position of unstable equilibrium.

20. Current loop as a magnetic dipole. A planar current loop of area A and carrying current I behaves as a magnetic dipole of dipole moment,

$$m = IA$$

In vector notation, $\vec{m} = I\vec{A}$

The direction of $\vec{m}$ is given by *right hand thumb rule*. If we curl the fingers of the right hand along the direction of current in the loop, then the extended thumb gives the direction of the magnetic moment associated with the loop.

21. Magnetic dipole moment of a revolving electron. The orbital magnetic moment of an electron revolving around a nucleus in n th orbit of radius r with speed v is given by

$$\mu_l = \frac{evr}{2} = \frac{e}{2m_e} l = n\left(\frac{eh}{4\pi m_e}\right)$$

where l is the magnitude of the angular momentum of the electron revolving around the nucleus.

22. Bohr magneton. It is the magnetic dipole moment associated with an electron due to its orbital motion in the first orbit of hydrogen atom. It is the smallest value of μ_l.

$$\mu_B = (\mu_l)_{\min} = \frac{eh}{4\pi m_e}$$

$$= 9.27 \times 10^{-24}\ \text{Am}^2$$

23. Gauss's law in magnetism. This law states that the net magnetic flux through any closed surface is zero.

Mathematically,

$$\phi_B = \oint_S \vec{B} \cdot d\vec{S} = 0.$$

This law indicates that isolated magnetic poles (also called monopoles) do not exist.

24. Basic difference between electric and magnetic lines of force. The magnetic lines of force are continuous and form closed loops. They do not start or end at a point. In contrast, the electric lines of force start from a positive charge and end on a negative charge or they fade out at infinity.

25. Earth's magnetic field. When a magnet is suspended freely, it orients itself roughly in the geographical north-south direction. This suggests that the earth behaves as a huge magnet. Its field can be approximated to that of a magnetic dipole with dipole moment $8.0 \times 10^{22}\ \text{Am}^2$ with its axis aligned at a small angle with the rotation axis of the earth. The magnitude of the field on the earth's surface is typically about 10^{-4} T.

26. Magnetic meridian. The vertical plane passing through the magnetic axis of a freely suspended small magnet is called magnetic meridian. The earth's magnetic field acts in the direction of the magnetic meridian.

27. Geographic meridian. The vertical plane passing through the geographic north and south poles is called geographic meridian.

28. Elements of earth's magnetic field. The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field. These are declination, dip and horizontal component of earth's magnetic field.

29. Magnetic declination (α). It is the angle between the geographic meridian and the magnetic meridian at the given place.

30. Angle of dip (δ). It is the angle made by the earth's total magnetic field with the horizontal direction.

31. Horizontal component of earth's magnetic field. It is the component of the earth's total magnetic field B acting in the horizontal direction.

$$B_H = B\cos\delta$$

At the magnetic equator, $\delta = 0°$,

$$B_H = B\cos 0° = B$$

At the magnetic poles, $\delta = 90°$,

$$B_H = B\cos 90° = 0.$$

32. Relations between the elements of earth's magnetic field.

$$B_H = B\cos\delta$$

and $$B_V = B\sin\delta$$

$$\frac{B_V}{B_H} = \tan\delta$$

and $$B = \sqrt{B_H^2 + B_V^2}$$

33. Neutral point. It is the point where the magnetic field of a bar magnet is completely cancelled by the horizontal component of earth's magnetic field.

(*i*) For a magnet placed with its N-pole pointing geographic north, the neutral points lie at the equatorial line.

At each neutral point,

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}}$$

$$\simeq \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} \quad \text{[For a short magnet]}$$

(*ii*) For a magnet placed with its N-pole pointing geographic south, the neutral points lie at the axial line. At each neutral point,

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{2\,mr}{(r^2 - l^2)^2}$$

$$\simeq \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} \quad \text{[For a short magnet]}$$

34. Magnetising field. The magnetic field that exists in vacuum and induces magnetism is called magnetising field. The magnetising field set up in a solenoid carrying current I and placed in vacuum,

$$B_0 = \mu_0\, nI$$

SI unit of B_0 is tesla (T).

35. Magnetising field intensity or magnetic intensity. It is the number of ampere-turns (nI) flowing round the unit length of the solenoid required to produce a given magnetising field.

Thus $H = nI$

Also, $B_0 = \mu_0\, nI = \mu H$

$\therefore \quad H = \frac{B_0}{\mu_0}$

SI unit of H is Am^{-1} and its dimensions are $[L^{-1}A]$.

36. Magnetisation or intensity of magnetisation. It is the magnetic moment developed per unit volume of a material when placed in a magnetising field. It is a vector quantity.

$$M = \frac{m}{V}$$

Its SI unit is Am^{-1} or $N\,Wb^{-1}$ or $Nm^{-2}\,T^{-1}$.

37. Magnetic induction. It is the total number of magnetic lines of force crossing per unit area through a magnetic material. Its SI unit is tesla (T).

$$B = \mu_0\,(H + M)$$

38. Magnetic permeability. It is the ratio of the magnetic induction to the magnetising field intensity.

$$\mu = \frac{B}{H}$$

Its SI unit is $Tm\,A^{-1}$ or $Wbm^{-1}\,A^{-1}$.

39. Relative permeability. It is the ratio of the permeability of the material to the permeability of free space.

$$\mu_r = \frac{\mu}{\mu_0}$$

40. Magnetic susceptibility. It is the ratio of the intensity of magnetisation (M) induced to the magnetising field intensity (H).

$$\chi_m = \frac{M}{H}$$

It can be shown that

$$\mu = \mu_0\,(1 + \chi_m)$$

and $\quad \mu_r = 1 + \chi_m.$

41. Classification of magnetic materials. Magnetic materials are broadly classified as diamagnetic, paramagnetic and ferromagnetic.

42. Diamagnetic substances. These are the substances which when placed in a magnetising field get feebly magnetised in the opposite direction of the applied field. Such substances are feebly repelled by magnets and tend to move slowly from stronger to weaker parts of a magnetic field.

Examples are Bi, Cu, Pb, Si, N_2 (at STP), H_2O and NaCl.

43. Paramagnetic substances. These are the substances which when placed in a magnetising field get feebly magnetised in the direction of the magnetising field. Such substances are feebly attracted by magnets and tend to move slowly from weaker to stronger parts of a magnetic field. Examples are Al, Na, Ca, O_2 (at STP) and $CuCl_2$.

44. Ferromagnetic substances. These are the substances which when placed in a magnetising field get strongly magnetised in the direction of the magnetising field. Such substances tend to move quickly from weaker to stronger parts of a field. Examples are Fe, Ni, Co, Gd, etc.

The magnetic susceptibility χ is small and negative for diamagnetic substances, small and positive for paramagnetic substance and large and positive for ferromagnetic substances. The relative permeability ($\mu_r = 1 + \chi$) is slightly less than 1 for diamagnetic substances, slightly greater than 1 for paramagnetic substances and of the order of thousands for ferromagnetic substances.

45. Curie's law. The magnetic susceptibility of a paramagnetic substance varies inversely with its absolute temperature.

$$\chi_m \propto \frac{1}{T}$$

or $$\chi_m = \frac{C}{T},$$

where C is curie constant.

46. Curie temperature. The temperature above which a ferromagnetic substance becomes paramagnetic is called Curie temperature (T_C). The modified Curie law for ferromagnetic substances above the Curie temperature is

$$\chi_m = \frac{C'}{T - T_C} \qquad (T > T_C)$$

This relation is also called *Curie-Weiss law*.

47. Hysteresis. The phenomenon of lagging of the magnetic induction behind the magnetising field in a ferromagnetic material is called hysteresis. The area of the hysteresis (B-H) loop gives the energy wasted in a sample when it is taken through a cycle of magnetisation.

48. Retentivity or remanence. The magnetic induction left behind in the sample after the magnetising field has been removed is called retentivity.

49. Coercivity. The value of the reverse magnetising field required to make the residual magnetism of a sample equal to zero is called coercivity.

50. Tangent law. When a short magnet is suspended freely under the combined action of two uniform perpendicular magnetic fields B and B_H, the magnet comes to rest making an angle θ with the direction of B_H such that $B = B_H \tan\theta$.

51. Tangent galvanometer. Its working is based on tangent law and is used to measure very small currents. It consists of a circular coil of radius R and N turns. If a current I in the coil produces deflection θ in the compass needle at the centre of the coil, then

$$I = \left(\frac{2\,R\,B_H}{\mu_0\,N}\right)\tan\theta = K\tan\theta$$

where $K = \dfrac{2\,R\,B_H}{\mu_0\,N}$, is the *reduction factor* of the tangent galvanometer.

52. Vibration magnetometer. It is used to compare magnetic moments of two bar magnets or to determine the horizontal component of earth's magnetic field.

If a bar magnet of magnetic moment m and moment of inertia I oscillates under the horizontal component (B_H) of earth's magnetic field, then its period of vibration is

$$T = 2\pi\sqrt{\frac{I}{m\,B_H}}.$$

53. Uses of vibration magnetometer.

(*i*) Measurement of magnetic moment of a magnet,

$$m = \frac{4\pi^2 I}{B_H T^2}$$

(*ii*) Comparison of magnetic moments of two magnets of same size and same mass.

$$\frac{m_2}{m_1} = \frac{T_1^2}{T_2^2}$$

(*iii*) Comparison of magnetic moments of two magnets of unequal sizes and masses.

$$\frac{m_1}{m_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$$

(*iv*) Comparison of horizontal component of earth's magnetic field.

$$\frac{B_H}{B_H'} = \frac{T'^2}{T^2}$$